CS406: Compilers

Spring 2022

Week 11: Loop Optimization, ..

Optimize Loops

- Example - Code Motion

Should be careful while doing optimization of loops

```
while J > I loop
   A(j) := 10/I;
   j := j + 2;
end loop;
```

Optimize Loops – Code Motion

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while J > I loop
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   j := j + 2;
end loop;
```

- Optimization: can move 10/I out of loop.

Optimize Loops – Code Motion

- Should be careful while doing optimization of loops

```
while J > I loop
   A(j) := 10/I;
   j := j + 2;
end loop;
```

- Optimization: can move 10/I out of loop
- What if I = 0?

Optimize Loops – Code Motion

- Should be careful while doing optimization of loops

```
while J > I loop
   A(j) := 10/I;
   j := j + 2;
end loop;
```

- Optimization: can move 10/I out of loop
- What if I = 0?
- What if I != 0 but loop executes zero times?

Optimization Criteria - Safety and Profitability

- Safety - is the code produced after optimization producing same result?
- Profitability - is the code produced after optimization running faster or uses less memory or triggers lesser number of page faults etc.

```
while J > I loop
   A(j) := 10/I;
   j := j + 2;
end loop;
```

- E.g. moving I out of the loop introduces exception (when I=0)
- E.g. if the loop is executed zero times, moving `A(j) := 10/I` out is not profitable

Optimize Loops – Code Generation

- The outline of code generation for ‘for’ loops looked like this:

```
for (<init_stmt>;<bool_expr>;<incr_stmt>)
  <stmt_list>
end
```

```
   <init_stmt>
LOOP:
   <bool_expr>
   j<!op> OUT
   <stmt_list>
INCR:
   <incr_stmt>
   jmp LOOP
OUT:
```

```
for (i=0; i<=255;i++) {
    <stmt_list>
}
```

Naïve code generation

```
        code for i=0;
LOOP:   code for i<=255
        jump0 OUT
        code for <stmt_list>
INCR:   code for i++
        jump LOOP
OUT:
```

Question: why naïve is not good?

Optimize Loops – Code Generation

- What happens when ub is set to the maximum possible integer representable by the type of `i`?

```
for (i=0; i<=255;i++) {
    <stmt_list>
}
```

Better code:

```
        code for i=0;
        code for lb=1, ub=255
        code for lb<=ub
        jump0 OUT
LOOP:   code for <stmt_list>
        code for lb=ub
        jump1 OUT
INCR:   code for i++
        jump LOOP
OUT:
```

generalizing:

```
        code for i=0;
        compute lb, ub
        code for lb<=ub
        jump0 OUT
        assign index=lb
        assign limit=ub
LOOP:   code for <stmt_list>
        code for index=limit
        jump1 OUT
INCR:   code for increment index
        jump LOOP
OUT:
```

Optimize Loops -Identifying Invariant Expressions

- How do we identify expressions that can be moved out of the loop?
  - `LoopDef = {}` set of variables defined (i.e. whose values are overwritten) in the loop body
  - `LoopUse = { }` 'relevant' variables used in computing an expression

```
Mark_Invariants(Loop L) {
   1. Compute LoopDef for L
   2. Mark as invariant all expressions,
      whose relevant variables don't belong
      to LoopDef
}
```

Optimize Loops -Identifying Invariant Expressions

- Example

LoopDef{}

```
for I = 1 to 100                →  {A, J, K, I}
   for J = 1 to 100             →  {A, J, K}
      for K = 1 to 100          →  {A, K}
         A[I][J][K] = (I*J)*K
```

Optimize Loops -Identifying Invariant Expressions

- Example

LoopUse{}

```
for I = 1 to 100              → {}
   for J = 1 to 100           → {I}
      for K = 1 to 100        → {I,J}
         A[I][J][K] = (I*J)*K
```

Optimize Loops -Identifying Invariant Expressions

- Example

Invariant Expressions

```
for I = 1 to 100
   for J = 1 to 100
      for K = 1 to 100          → { I*J, Addr(A[i][j])}
         A[I][J][K] = (I*J)*K
```

For an array access, `A[m] => Addr(A) + m`

For 3D array above*, `Addr(A[I][J][K])` =

`Addr(A)+(I*10000)-10000+(J*100)-100+K-1`

*Assuming row-major ordering of storage

Optimize Loops -Identifying Invariant Expressions

- Example

```
for I = 1 to 100
   for J = 1 to 100
      for K = 1 to 100
         A[I][J][K] = (I*J)*K
```

Invariant Expressions

{ Addr(A[i]) }

For an array access, A[m] => Addr(A) + m

For 3D array above*, Addr(A[I][J][K]) =

Addr(A)+(I*10000)-10000+(J*100)-100+K-1

*Assuming row-major ordering of storage

Optimize Loops -Factoring Invariant Expressions

- Move the invariant expressions identified

```
Factor_Invariants(Loop L) {
   Mark_Invariants(L);
   foreach expression E marked an invariant:
      1. Create a temporary T
      2. Replace each occurrence of E in L with T
      3. Insert T:=E in L’s header code
         // If loop is known to execute at least once,
         insert T:=E before LOOP:
}
```

Optimize Loops -Factoring Invariant Expressions

- Example

```
for I = 1 to 100
   for J = 1 to 100
      for K = 1 to 100
         A[I][J][K] = (I*J)*K        //Invariant Expressions
```

Optimize Loops -Factoring Invariant Expressions

- Example

```
for I = 1 to 100
   for J = 1 to 100
      temp1=A[I][J]
      temp2=I*J
      for K = 1 to 100
         temp1[K] = temp2*K
```

Optimize Loops -Factoring Invariant Expressions

- Example

```
for I = 1 to 100
  temp3=A[I]
  for J = 1 to 100
    temp1=temp3[J]
    temp2=I*J
    for K = 1 to 100
      temp1[K] = temp2*K
```

Optimize Loops -Factoring Invariant Expressions

- Expressions cannot always be moved out!

Case I: We can move `t = a op b` if the statement dominates all loop exits where `t` is live

A node bb1 dominates node bb2 if all paths to bb2 must go through bb1

```
for (...) {
   if(*)
      a = 100
}
c=a
```

Cannot move `a=100` because it does not dominate `c=a` i.e. there is one path (when if condition is false) `c=a` can be executed /'reached' without going to `a=100`

Optimize Loops -Factoring Invariant Expressions

- Expressions cannot always be moved out!

  Case II: We can move `t = a op b` if there is only one definition of `t` in the loop

```
for (...) {
   if(*)
      a = 100
   else
      a = 200
}
```

Multiple definition of a

Optimize Loops -Factoring Invariant Expressions

- Expressions cannot always be moved out!

Case III: We can move `t = a op b` if `t` is not defined before the loop, where the definition reaches t's use after the loop

```
a=5
for (...) {
   a = 4+b
}
c=a
```

Definition of a in a=5 reaches `c=a,` which is defined after the loop

Optimize Loops –Strength Reduction

- Like strength reduction in peephole optimization
  - E.g. replace `a*2` with `a<<1`
- Applies to uses of induction variable in loops
  - Basic induction variable (`I`) – only definition within the loop is of the form `I = I ± S`, (S is loop invariant)

    `I` *usually determines number of iterations*
  - Mutual induction variable (`J`) – defined within the loop, its value is linear function of other induction variable, `I`, such that

    `J = I * C ± D` (C, D are loop invariants)

Optimize Loops –Strength Reduction

```
strength_reduce(Loop L) {
   Mark_Invariants(L);
   foreach expression E of the form I*C+D where I is
L's loop index and C and D are loop invariants
      1. Create a temporary T
      2. Replace each occurrence of E in L with T
      3. Insert T:=Io*C+D, where Io is the initial value of the
         induction variable, immediately before L
      4. Insert T:=T+S*C, where S is the step size, at the end of
         L's body
      }
```

Optimize Loops –Strength Reduction

- Suppose induction variable `I` takes on values I_o, I_o+S, I_o+2S, I_o+3S... in iterations 1, 2, 3, 4, and so on…
- Then, in consecutive iterations, Expression `I*C+D` takes on values

$$I_o*C+D$$
$$(I_o+S)*C+D = I_o*C+S*C+D$$
$$(I_o+2S)*C+D = I_o*C+2S*C+D$$
$$\ldots \qquad \ldots$$

- The expression changes by a constant `S*C`
- Therefore, we have replaced a * and + with a +

Optimize Loops – Strength Reduction

- Example (Applying to innermost loop)

```
for I = 1 to 100
   for J = 1 to 100
      for K = 1 to 100
         A[I][J][K] = (I*J)*K
```

→

```
for I=1 to 100
   temp3=Addr(A[i])
   for J=1 to 100
      temp1=Addr(temp3(J))
      temp2=I*J
      for K=1 to 100
         temp1[K]=temp2*K
```

↲

```
. . .
temp2=I*J
temp4=temp2
for K=1 to 100
   temp1[K]=temp4
   temp4=temp4+temp2
```

```
//S=1
//C=temp2
```

Optimize Loops – Strength Reduction

- Exercise (Apply to intermediate loop)

```
for I=1 to 100
   temp3=Addr(A[i])
   for J=1 to 100
      temp1=Addr(temp3(J))
      temp2=I*J
      for K=1 to 100
         temp1[K]=temp2*K
```

➡

```
. . .
temp2=I*J
temp4=temp2
for K=1 to 100
   temp1[K]=temp4
   temp4=temp4+temp2
```

```
// Induction var = J
// S = 1
// Expression = I * J
```

Optimize Loops – Strength Reduction

- Exercise (Apply to intermediate loop)

… → …

```
. . .
temp5=I
for J=1 to 100
      temp1=Addr(temp3(J))
      temp2=temp5
      temp4=temp2
      for K=1 to 100
         temp1[K]=temp4
         temp4=temp4+temp2
      temp5=temp5+I
```

Optimize Loops – Strength Reduction

- Further strength reduction possible?

```
for I=1 to 100
   temp3=Addr(A[i])
   temp5=I
   for J=1 to 100
      temp1=Addr(temp3(J))
      temp2=temp5
      temp4=temp2
      for K=1 to 100
         temp1[K]=temp4
         temp4=temp4+temp2
      temp5=temp5+I
```

Optimize Loops – Loop Unrolling

- Modifying induction variable in each iteration can be expensive
- Can instead *unroll* loops and perform multiple iterations for each increment of the induction variable
- What are the advantages and disadvantages?

```
for (i = 0; i < N; i++)
  A[i] = ...
```

↓ Unroll by factor of 4

```
for (i = 0; i < N; i += 4)
  A[i] = ...
  A[i+1] = ...
  A[i+2] = ...
  A[i+3] = ...
```

Optimize Loops - Summary

- Low level optimization
  - Moving code around in a single loop
  - Examples: loop invariant code motion, strength reduction, loop unrolling
- High level optimization
  - Restructuring loops, often affects multiple loops
  - Examples: loop fusion, loop interchange, loop tiling

Useful optimizations

- Common subexpression elimination (global)
  - Need to know which expressions are available at a point
- Dead code elimination
  - Need to know if the effects of a piece of code are never needed, or if code cannot be reached
- Constant folding
  - Need to know if variable has a constant value
- So how do we get this information?

Dataflow analysis

- Framework for doing compiler analyses to drive optimization
- Works across basic blocks
- Examples
  - Constant propagation: determine which variables are constant
  - Liveness analysis: determine which variables are live
  - Available expressions: determine which expressions have valid computed values
  - Reaching definitions: determine which definitions could "reach" a use

Slide Courtesy: Milind Kulkarni

Dataflow Analysis - Common Traits

Common requirement among global optimizations:

- Know a particular **property X** at a *program point*

  (There is a program point one before a statement and one after a statement)

  - Say that property X definitely holds.

    OR

  - Don't know if property X holds or not (okay to be conservative)

This requires the knowledge of entire program

Dataflow analysis

- Framework for doing compiler analyses to drive optimization
- Works across basic blocks
- Examples
    - Constant propagation: determine which variables are constant
    - Liveness analysis: determine which variables are live
    - Available expressions: determine which expressions have valid computed values
    - Reaching definitions: determine which definitions could "reach" a use

Slide Courtesy: Milind Kulkarni

Liveness – Recap..

X **defined** here

```
1: X = 10
   …….
N: Y = X + 5
```

X **used** here

X is live at 1

..used in future

- A variable X is live at statement S if:
  - There is a statement S' that uses X
  - There is a path from S to S'
  - There are no intervening definitions of X

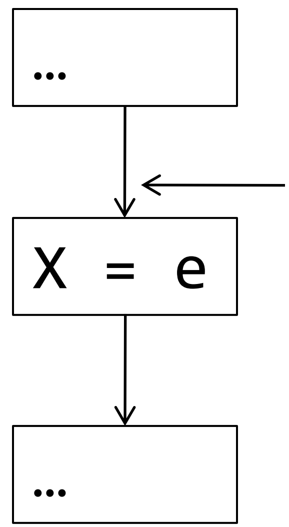
Liveness – Recap..

```
1: X = 10
2: X = Y + 2
   ….
N: Y = X + 5
```

X is <u>dead</u> at 1

- A variable X is dead at statement S if it is not live at S
  - What about …; `X = X + 1`?

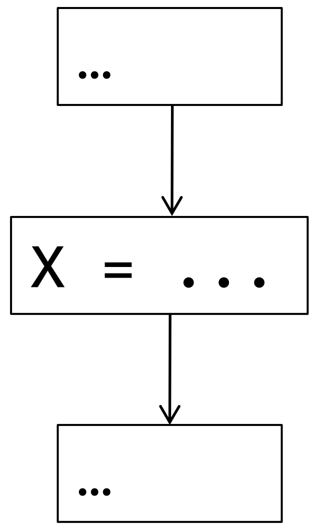
Liveness in a CFG



Given that `e` does not use `X`, `X` is *definitely dead* here (i.e. before the statement).

- Define a set `LiveIn(b)`, where b is a basic block, as: the set of all variables live at the entrance of a basic block

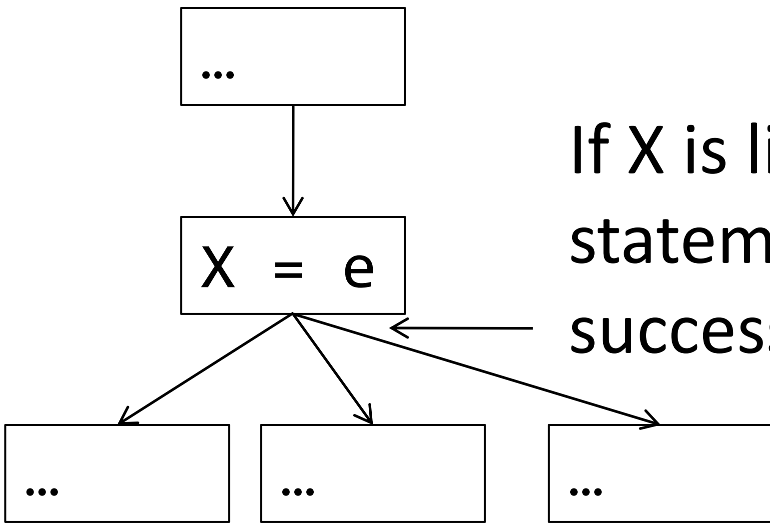
Liveness in a CFG



X is defined here

- Define a set `Def(b)`, where b is a basic block, as: the set of all variables that are defined in b

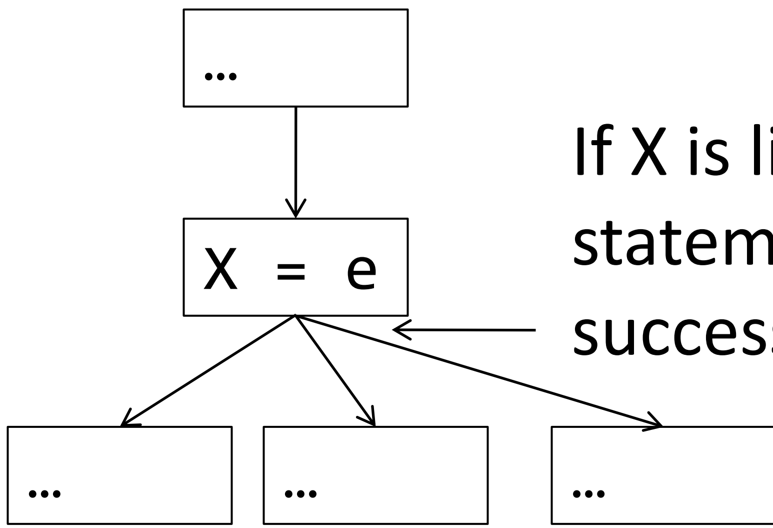
Liveness in a CFG



If X is live here (i.e. after the statement), X is used in *some* successor

- Define a set `LiveOut(b)`, where b is a basic block, as: the set of all variables live at the exit of a basic block

Liveness in a CFG

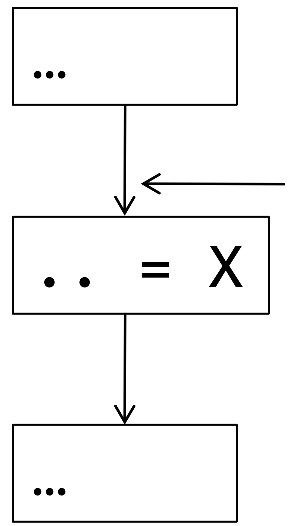


If X is live here (i.e. after the statement), X is used in *some* successor

- If S(b) is the set of all successors of b, then

$$\text{LiveOut}(b) = \bigcup_{i \in S(b)} \text{LiveIn}(i)$$

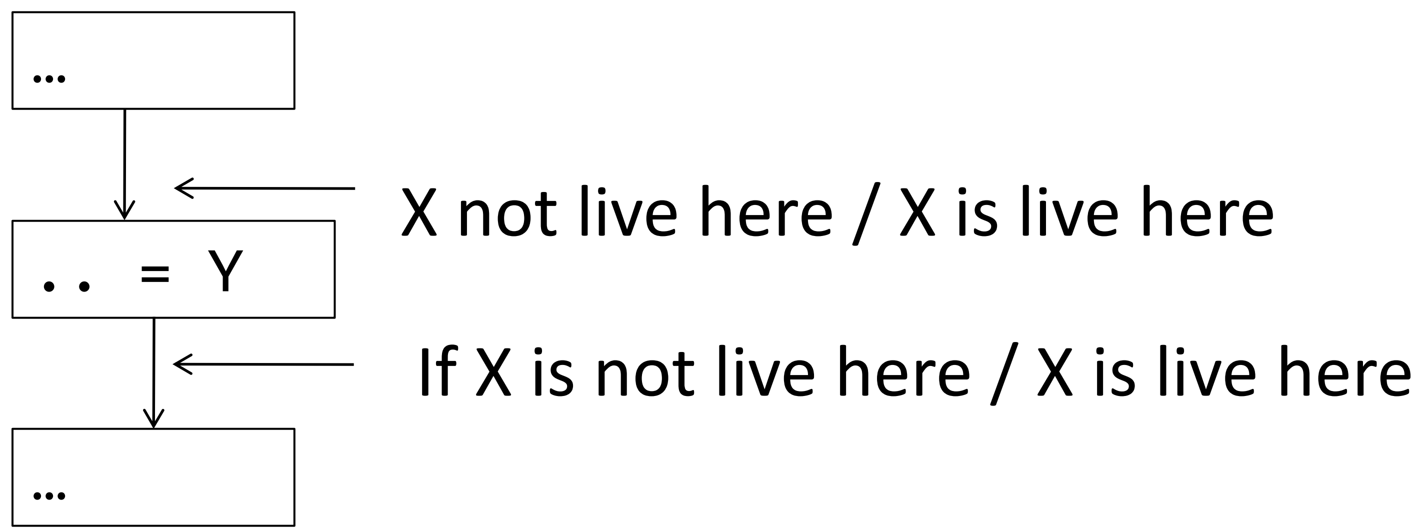
Liveness in a CFG



X *must be* live here (i.e. before the statement)

- Define a set `LiveUse(b)`, where b is a basic block, as the set of all variables that are used before they are defined within block `b`. `LiveIn(b)` $\supseteq$ `LiveUse(b)`

Liveness in a CFG - Observation



- If a node neither uses nor defines X, the liveness property remains the same before and after executing the node

Liveness in a CFG

- If a variable is live on exit from b, it is either defined in b or live on entrance to b

$$\text{LiveIn(b)} \supseteq \text{LiveOut(b)} - \text{Def(b)}$$

- Under what scenarios can a variable be live at the entrance of a basic block?

Liveness in a CFG

- If a variable is live on exit from b, it is either defined in b or live on entrance to b

$$\text{LiveIn(b)} \supseteq \text{LiveOut(b)} - \text{Def(b)}$$

- Under what scenarios can a variable be live at the entrance of a basic block?
  - Either the variable is used in the basic block

Liveness in a CFG

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$$\text{LiveIn}(b) \supseteq \text{LiveOut}(b) - \text{Def}(b)$$

- Under what scenarios can a variable be live at the entrance of a basic block?
  - Either the variable is used in the basic block
  - OR the variable is live at exit and not defined within the block

Liveness in a CFG

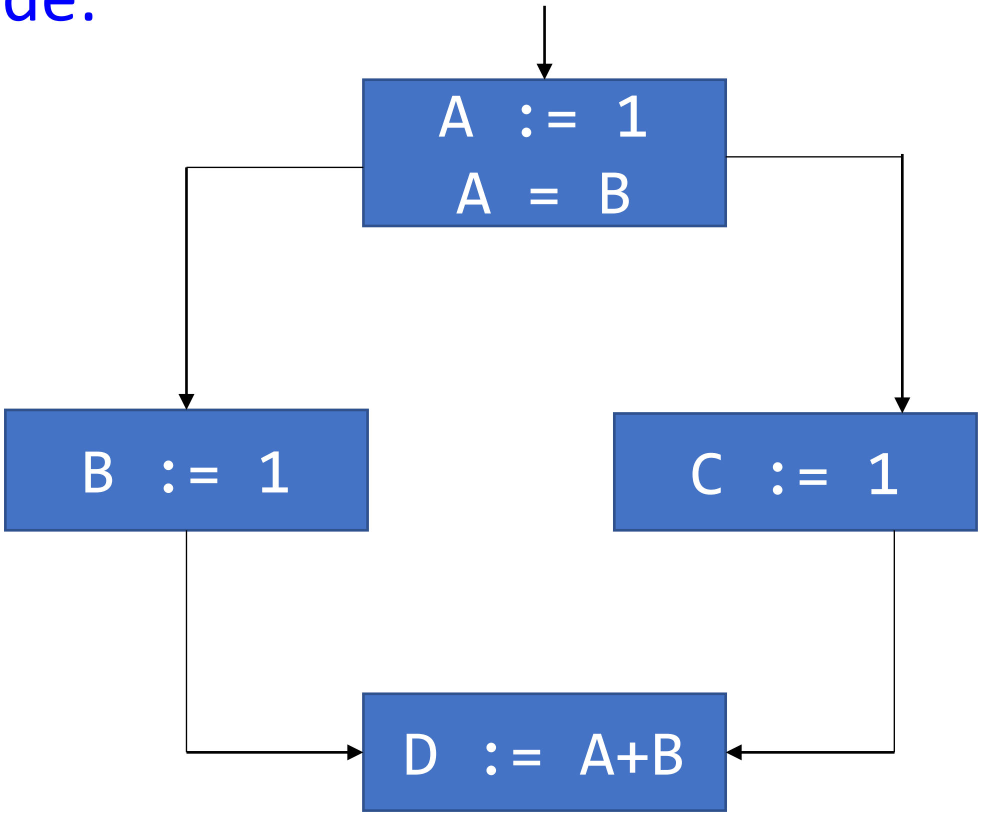
- Under what scenarios can a variable be live at the entrance of a basic block?
  - Either the variable is used in the basic block
  - OR the variable is live at exit and not defined within the block

```
LiveIn(b) = LiveUse(b) U  (LiveOut(b) –
Def(b))
```

Liveness in a CFG - Example

- Draw CFG for the code:

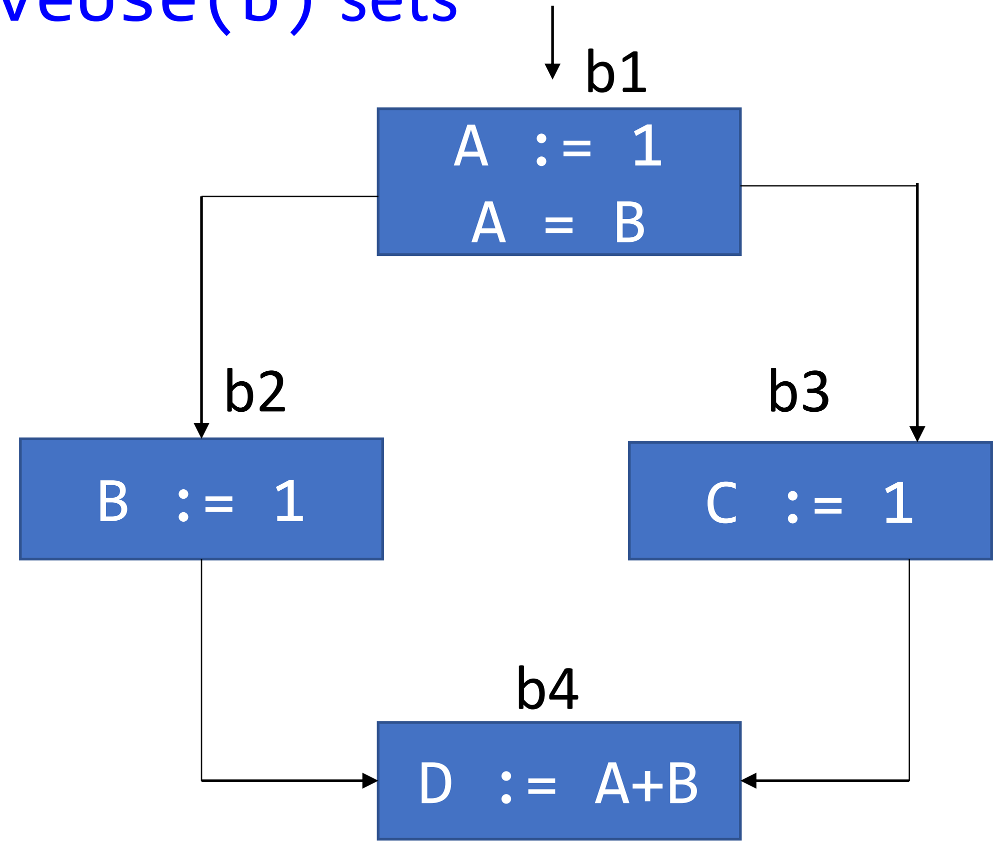
```
A:=1
if A=B then
   B:=1
else
   C:=1
endif
D:=A+B
```



Liveness in a CFG - Example

- Compute `Def(b)` and `LiveUse(b)` sets

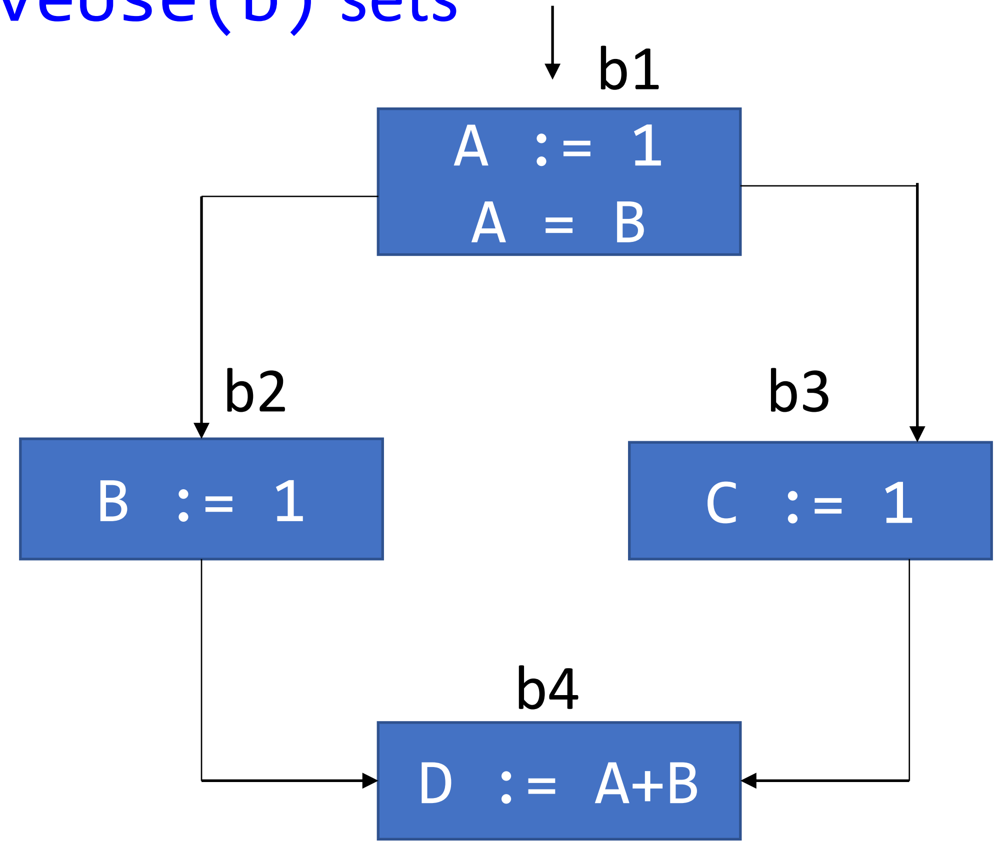
Block	Def	LiveUse
b1		
b2		
b3		
b4		



Liveness in a CFG - Example

- Compute `Def(b)` and `LiveUse(b)` sets

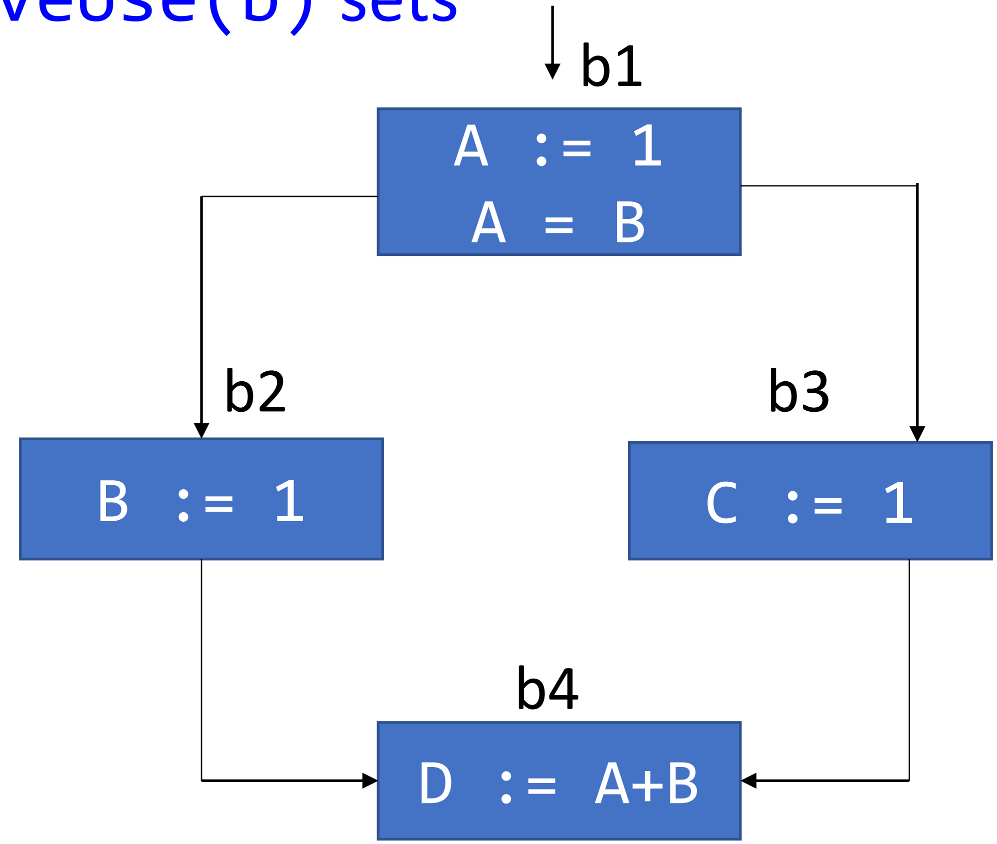
Block	Def	LiveUse
b1	{A}	{B}
b2		
b3		
b4		



Liveness in a CFG - Example

- Compute `Def(b)` and `LiveUse(b)` sets

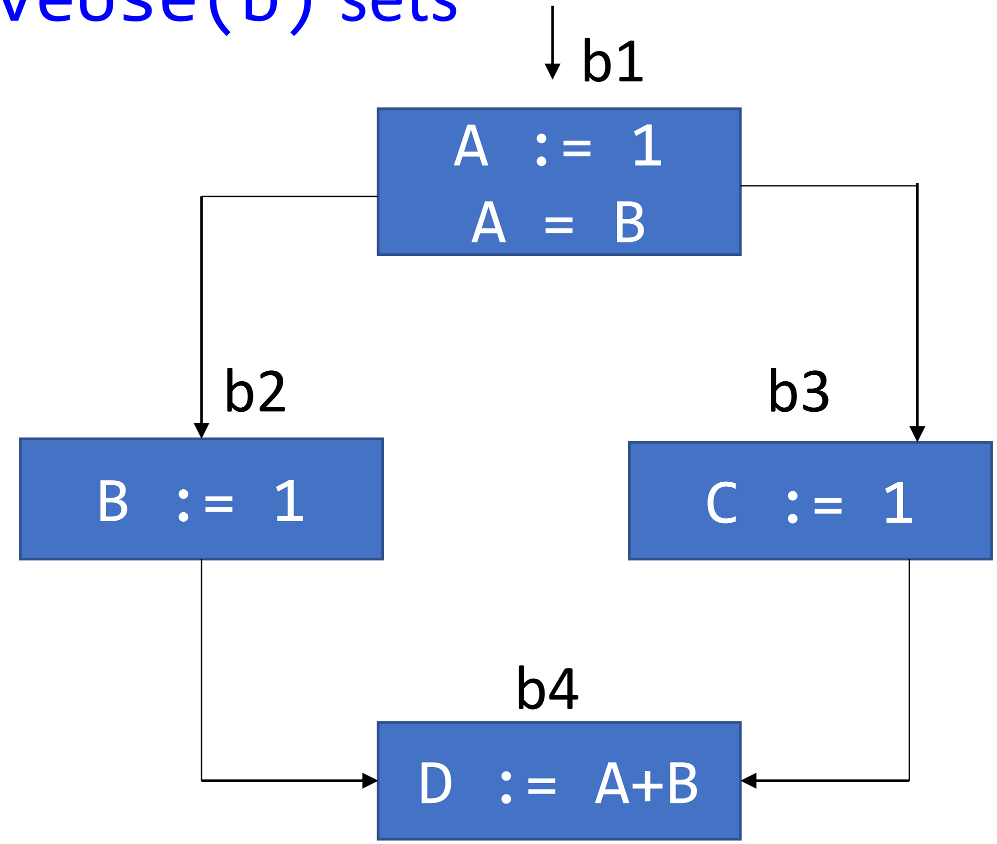
Block	Def	LiveUse
b1	{A}	{B}
b2	{B}	{}
b3		
b4		



Liveness in a CFG - Example

- Compute `Def(b)` and `LiveUse(b)` sets

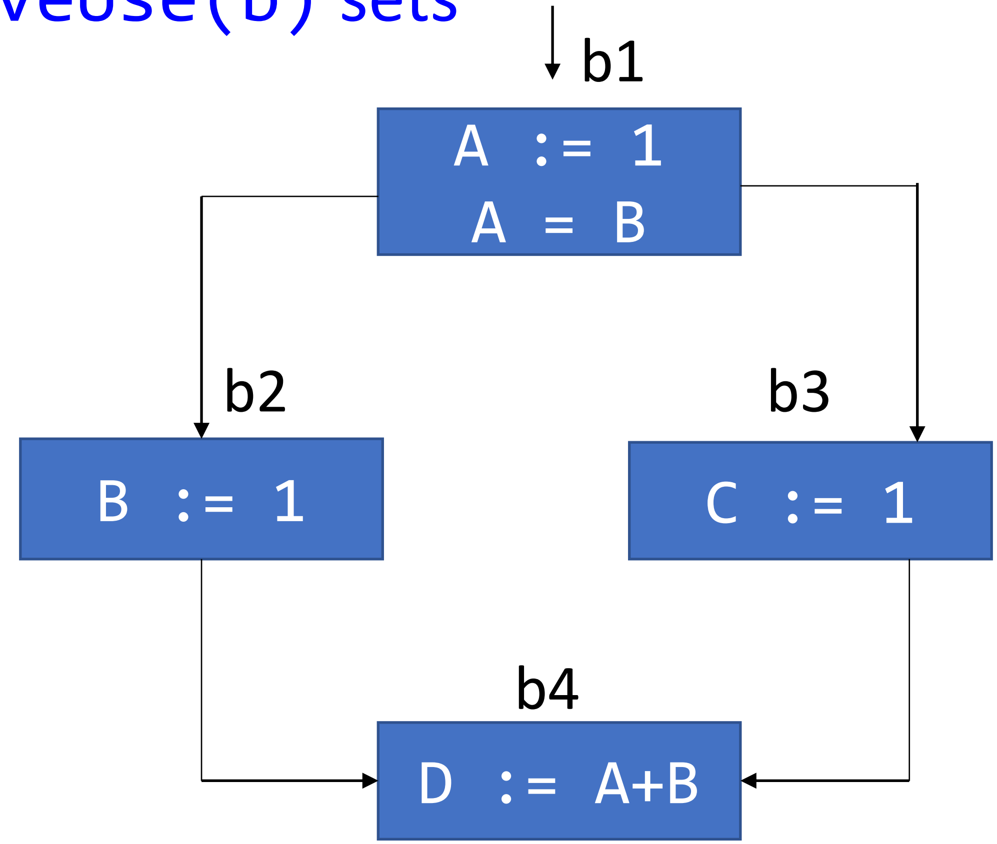
Block	Def	LiveUse
b1	{A}	{B}
b2	{B}	{}
b3	{C}	{}
b4		



Liveness in a CFG - Example

- Compute `Def(b)` and `LiveUse(b)` sets

Block	Def	LiveUse
b1	{A}	{B}
b2	{B}	{}
b3	{C}	{}
b4	{D}	{A,B}

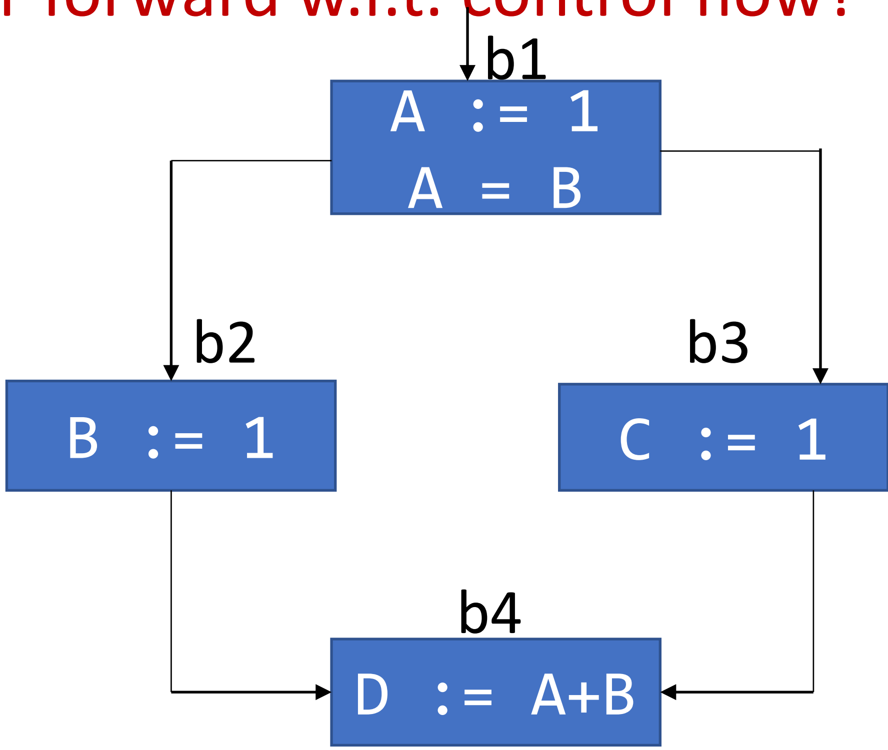


Liveness in a CFG - Example

- start from use of a variable to its definition.

Is this analysis going backward or forward w.r.t. control flow?

Block	Def	LiveUse
b1	{A}	{B}
b2	{B}	{}
b3	{C}	{}
b4	{D}	{A,B}

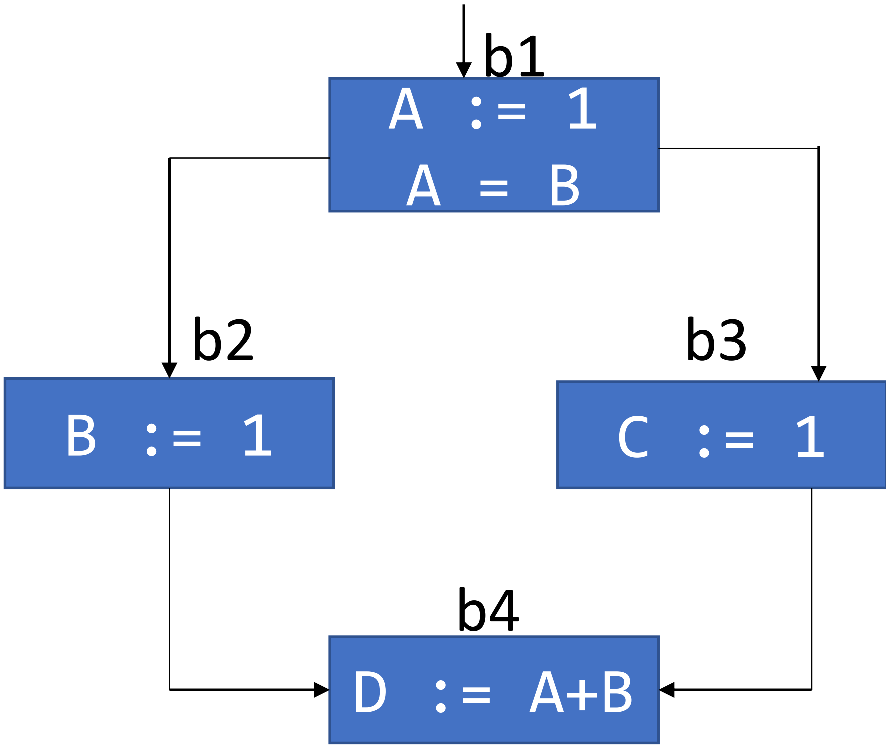


Liveness in a CFG - Example

- start from use of a variable to its definition.

Backward-flow problem

Block	Def	LiveUse
b1	{A}	{B}
b2	{B}	{}
b3	{C}	{}
b4	{D}	{A,B}

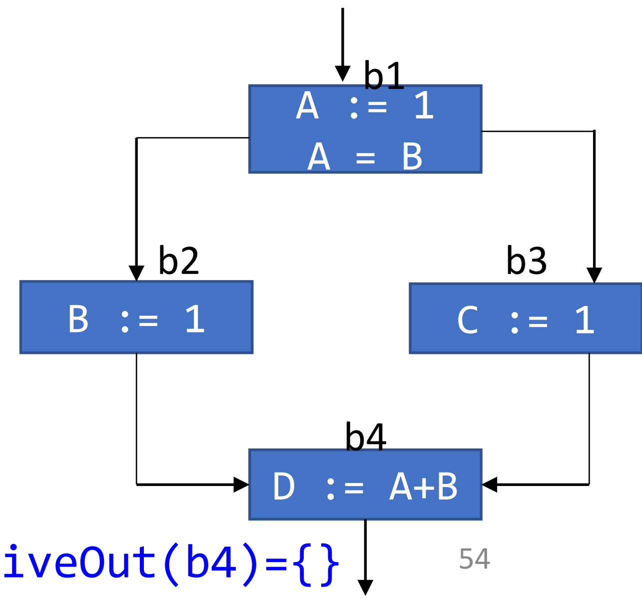


Liveness in a CFG - Example

- Start from use of a variable to its definition.
- Compute LiveOut and LiveIn sets:

$$\text{LiveIn(b)} = \text{LiveUse(b)} \cup (\text{LiveOut(b)} - \text{Def(b)})$$

Block	Def	LiveUse
b1	{A}	{B}
b2	{B}	{}
b3	{C}	{}
b4	{D}	{A,B}

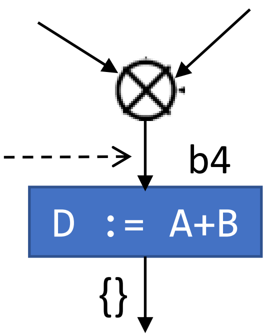


LiveOut(b4)={}

Liveness in a CFG - Example

```
LiveIn(b4) = LiveUse(b4) U  (LiveOut(b4) - Def(b4))
           = {A,B} U  ({} - {D})
```

Program point



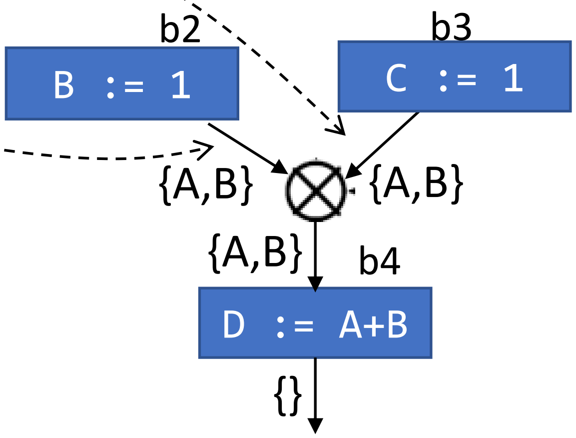
Block	Def	LiveUse
b1	{A}	{B}
b2	{B}	{}
b3	{C}	{}
b4	{D}	{A,B}

Liveness in a CFG - Example

LiveOut(b) = $\cup_{i \in S(b)}$ LiveIn(i)

LiveOut(b3) = LiveIn(b4) = {A,B}

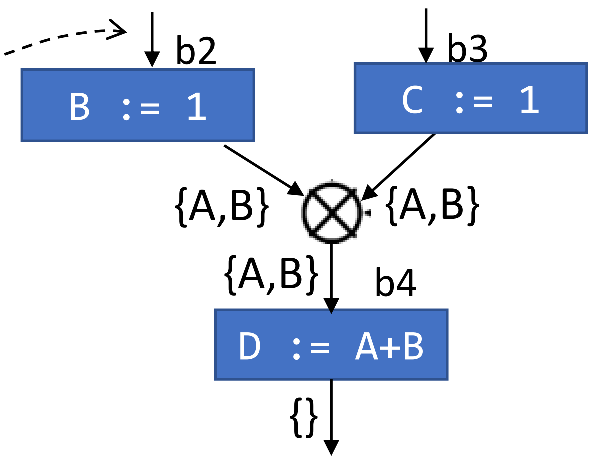
LiveOut(b2) = LiveIn(b4) = {A,B}



Block	Def	LiveUse
b1	{A}	{B}
b2	{B}	{}
b3	{C}	{}
b4	{D}	{A,B}

Liveness in a CFG - Example

```
LiveIn(b3) = LiveUse(b3) U  (LiveOut(b3) – Def(b3))
           = {} U  ({A,B} – {C}) = {A,B}
LiveIn(b2) = LiveUse(b2) U  (LiveOut(b2) – Def(b2))
           = {} U  ({A,B} – {B}) = {A}
```



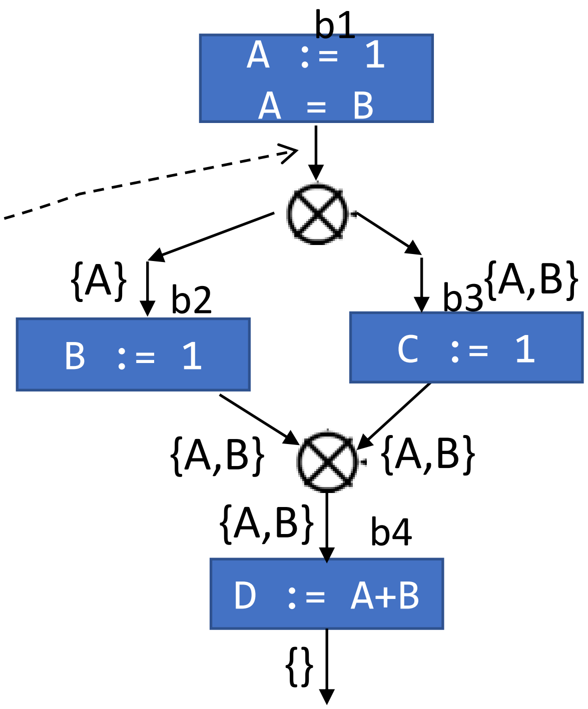
Block	Def	LiveUse
b1	{A}	{B}
b2	{B}	{}
b3	{C}	{}
b4	{D}	{A,B}

Liveness in a CFG - Example

$\texttt{LiveOut(b)} = \bigcup_{i \in S(b)} \text{LiveIn}(i)$

$\texttt{LiveOut(b1)} = \text{LiveIn}(b2) \cup \text{LiveIn}(b3)$

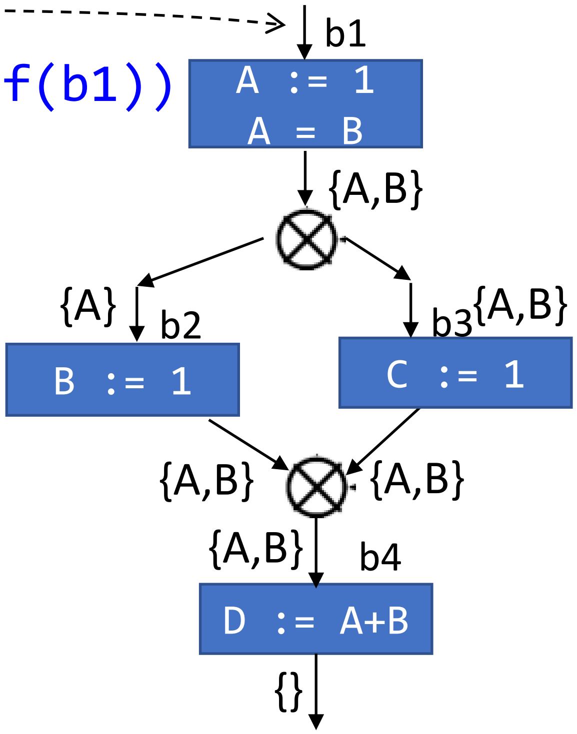
$= \{A\} \cup \{A,B\} = \{A,B\}$



Block	Def	LiveUse
b1	{A}	{B}
b2	{B}	{}
b3	{C}	{}
b4	{D}	{A,B}

Liveness in a CFG - Example

```
LiveIn(b1) = LiveUse(b1) U  (LiveOut(b1) – Def(b1))
           = {B} U  ({A,B} – {A}) = {B}
```



Block	Def	LiveUse
b1	{A}	{B}
b2	{B}	{}
b3	{C}	{}
b4	{D}	{A,B}

Liveness in a CFG - Example

- Summary: Compute `LiveIn(b)` and `LiveOut(b)`

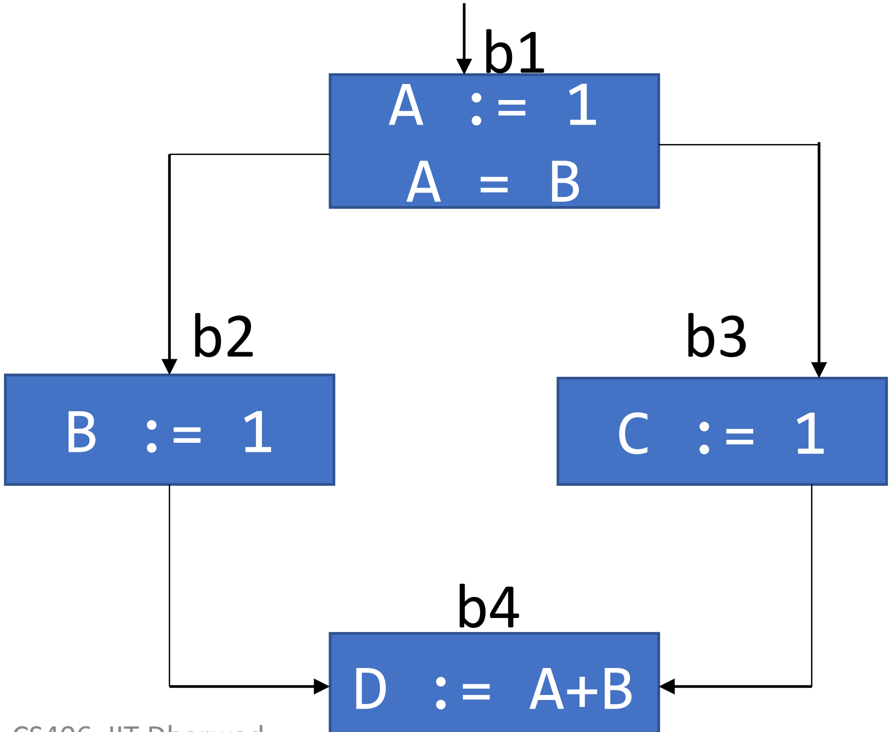
```
LiveIn(b) = LiveUse(b) U  (LiveOut(b) – Def(b))
```

Block	Def	LiveUse
b1	{A}	{B}
b2	{B}	{}
b3	{C}	{}
b4	{D}	{A,B}

Block	LiveIn	LiveOut
b1	{B}	{A,B}
b2	{A}	{A,B}
b3	{A,B}	{A,B}
b4	{A,B}	{}

Liveness in a CFG – Use Case

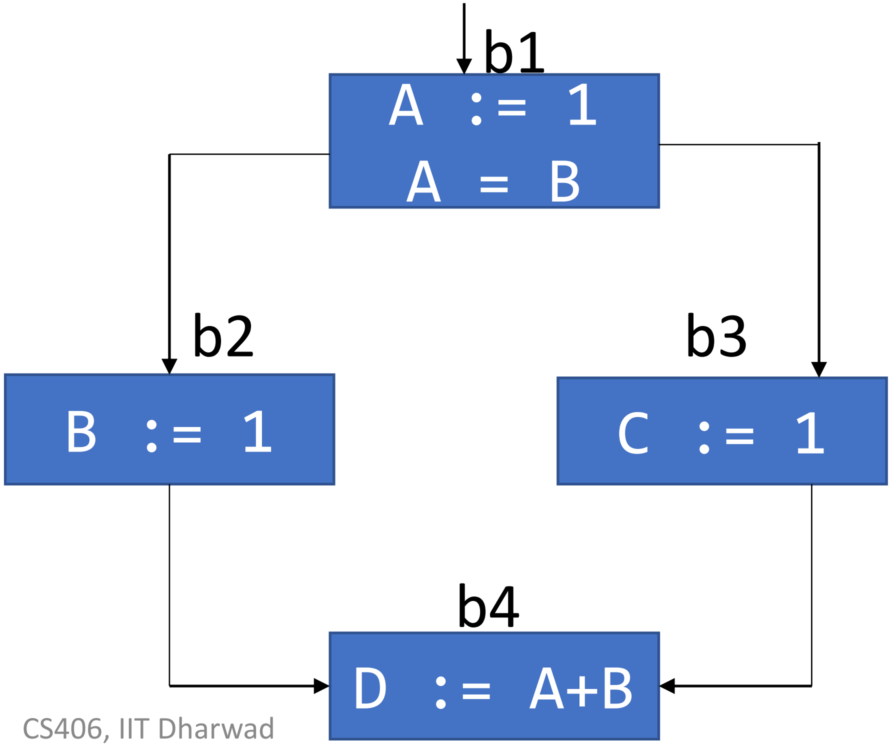
- Assume that the CFG below represents *your entire program* (b1 is the entry to program and b4 is the exit)
    - What can you infer from the table?



Block	LiveIn	LiveOut
b1	{B}	{A,B}
b2	{A}	{A,B}
b3	{A,B}	{A,B}
b4	{A,B}	{}

Liveness in a CFG – Use Case

- Assume that the CFG below represents *your entire program*
    - Variable B is live at the entrance of b1, the entry basic block of CFG. This implies that B is used before it is defined. An error!



Block	LiveIn	LiveOut
b1	{B}	{A,B}
b2	{A}	{A,B}
b3	{A,B}	{A,B}
b4	{A,B}	{}

Liveness in a CFG – Use Case

- Liveness information tells us what variable is dead. Can remove statements that assign to dead variables.

X is dead here implies that we can remove this statement.

```
X = 1
Y = X + 2
Z = Y + A
```

⇨

```
X = 1
Y = 1 + 2
Z = Y + A
```

⇨

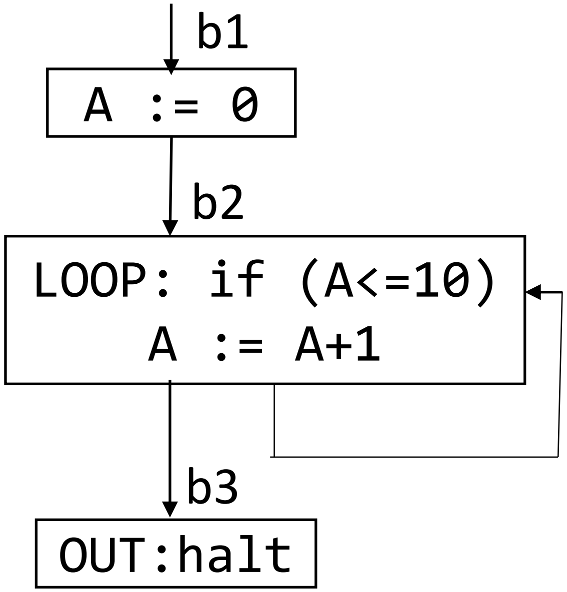
```
X = 1
Y = 1 + 2
Z = Y + A
```

Constant Propagation

Dead Code Elimination

Liveness in a CFG – Example (Loop)

- How do we compute liveness information when a loop is present?



Block	Def	LiveUse
b1	{A}	{}
b2	{A}	{A}
b3	{}	{}

Block	LiveIn	LiveOut
b1	{}	{A}
b2	{A}	{A}
B3	{}	{}

Liveness in a CFG - Observations

- Liveness is computed as information is *transferred* between adjacent statements
- At a program point, a variable can be live or not live (property: true or false)
    - To begin with we did not have any information=property is false

<u>At a program point</u> can the liveness information change?

- Yes, Liveness information changes from false to true and not otherwise.

How can we find constants?

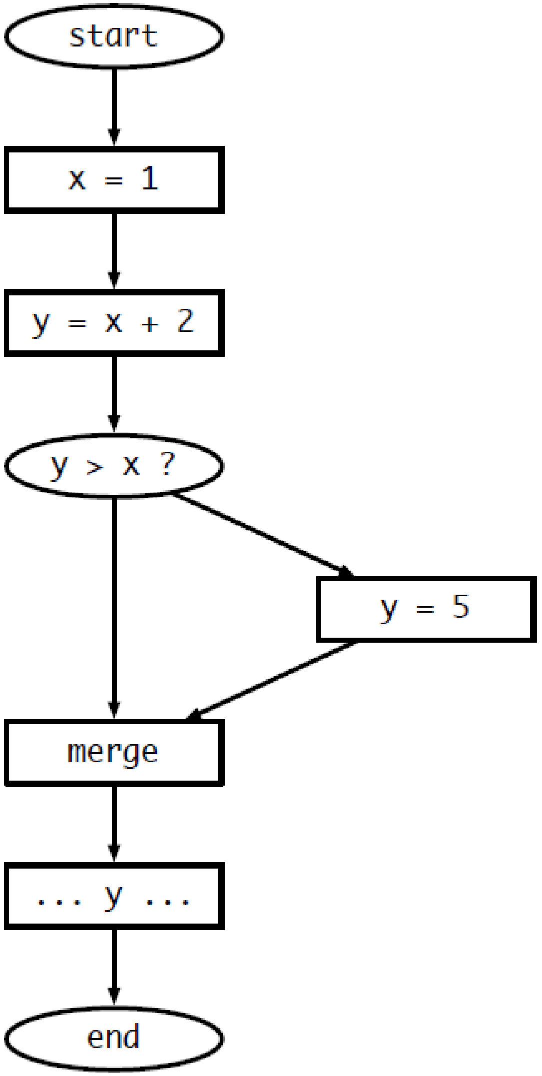
- Ideal: run program and see which variables are constant
  - Problem: variables can be constant with some inputs, not others – need an approach that works for all inputs!
  - Problem: program can run forever (infinite loops?) – need an approach that we know will finish
- Idea: run program *symbolically*
  - Essentially, keep track of whether a variable is constant or not constant (but nothing else)

Overview of algorithm

- Build control flow graph
  - We'll use statement-level CFG (with merge nodes) for this
- Perform symbolic evaluation
  - Keep track of whether variables are constant or not
- Replace constant-valued variable uses with their values, try to simplify expressions and control flow

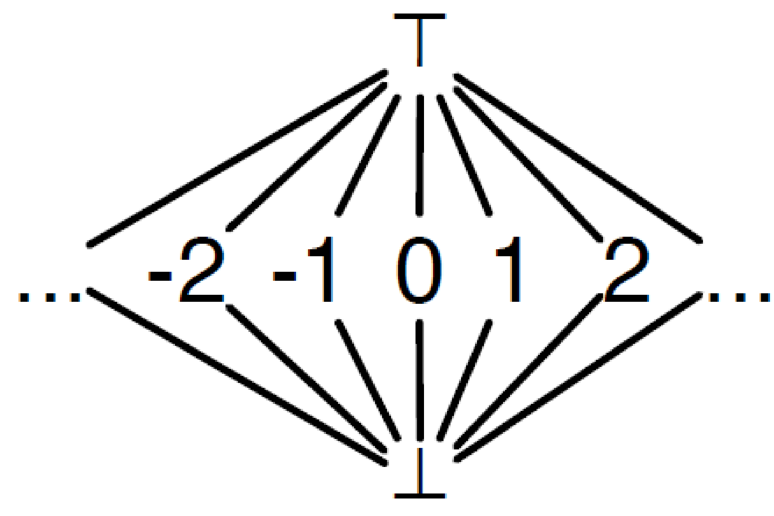
Build CFG

```
x = 1;
y = x + 2;
if (y > x) then y = 5;
... y ...
```



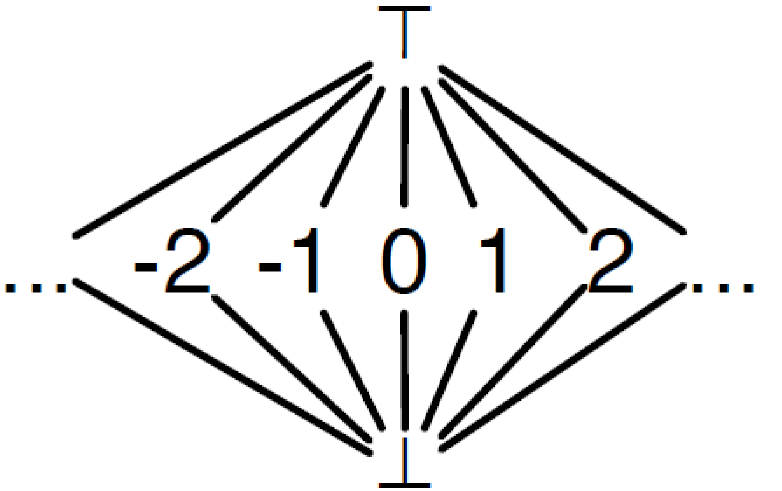
Symbolic evaluation

- Idea: replace each value with a symbol
  - constant (specify which), no information, definitely not constant
- Can organize these possible values in a *lattice*
  - Set of possible values, arranged from least information to most information



Symbolic evaluation

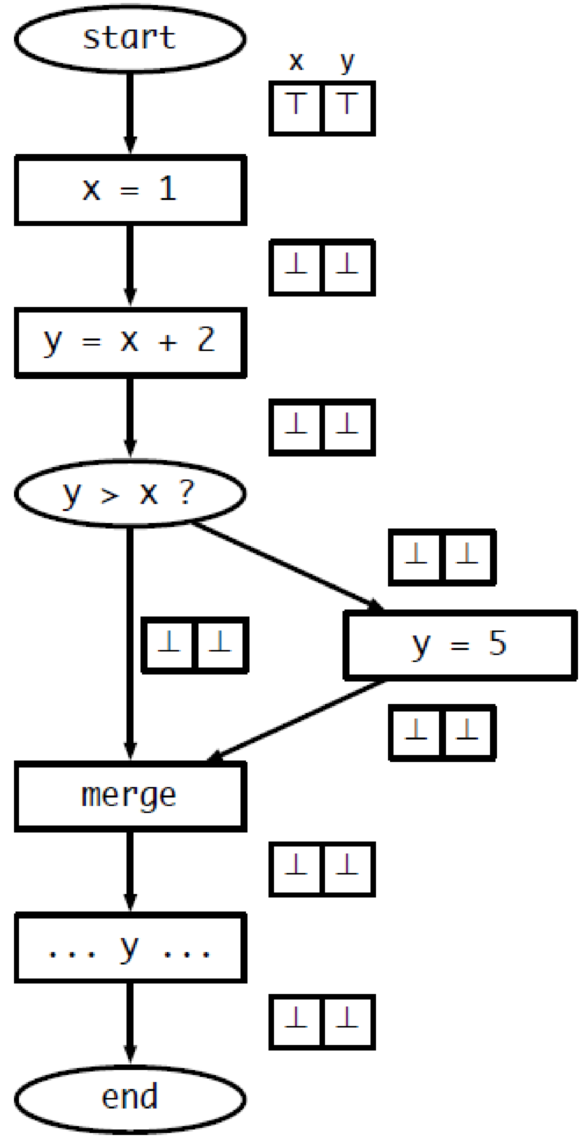
- Evaluate expressions symbolically: eval(e, V_{in})
  - If e evaluates to a constant, return that value. If any input is $\top$ (or $\bot$), return $\top$ (or $\bot$)
    - Why?
- Two special operations on lattice
  - meet(a, b) – highest value less than or equal to both a and b
  - join(a, b) – lowest value greater than or equal to both a and b



Join often written as a $\sqcup$ b
Meet often written as a $\sqcap$ b

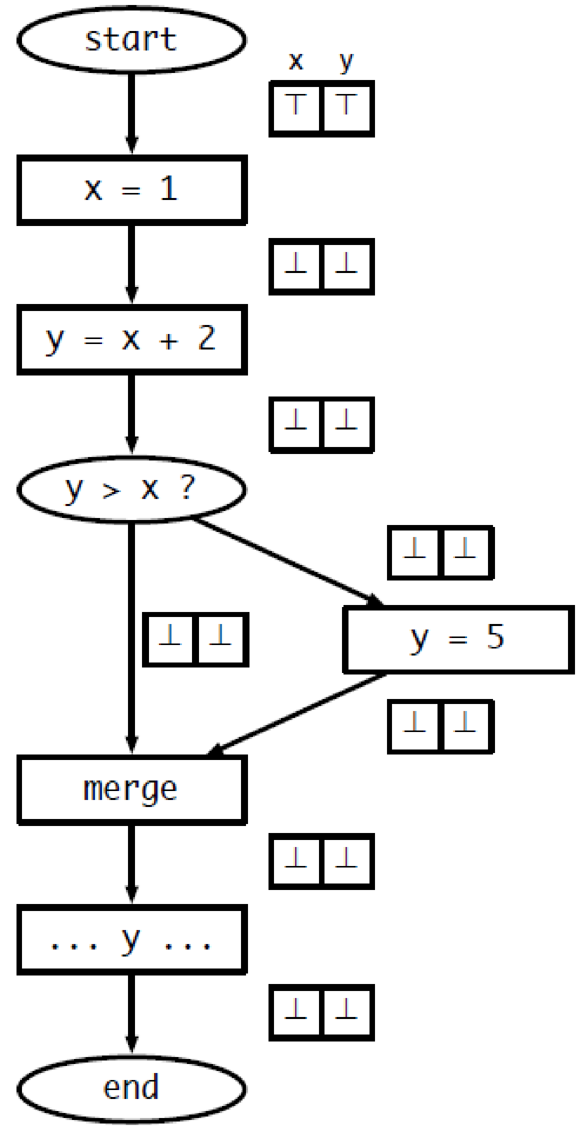
Putting it together

- Keep track of the symbolic value of a variable at every program point (on every CFG edge)
  - State vector
- What should our initial value be?
  - Starting state vector is all ⊤
    - Can't make any assumptions about inputs – must assume not constant
  - Everything else starts as ⊥, since we have no information about the variable at that point



Executing symbolically

- For each statement `t = e` evaluate `e` using V_{in}, update value for `t` and propagate state vector to next statement
- What about switches?
  - If `e` is true or false, propagate V_{in} to appropriate branch
  - What if we can't tell?
    - Propagate V_{in} to both branches, and symbolically execute both sides
- What do we do at merges?



Handling merges

- Have two different V_{in}s coming from two different paths
- Goal: want new value for V_{in} to be *safe* (shouldn't generate wrong information), and we don't know which path we actually took
- Consider a single variable. Several situations:
  - $V_1 = \bot, V_2 = * \rightarrow V_{out} = *$
  - V_1 = constant x, $V_2 = x \rightarrow V_{out} = x$
  - V_1 = constant x, V_2 = constant y $\rightarrow V_{out} = \top$
  - $V_1 = \top, V_2 = * \rightarrow V_{out} = \top$
- Generalization:
  - $V_{out} = V_1 \sqcup V_2$