

CS406: Compilers

Spring 2020

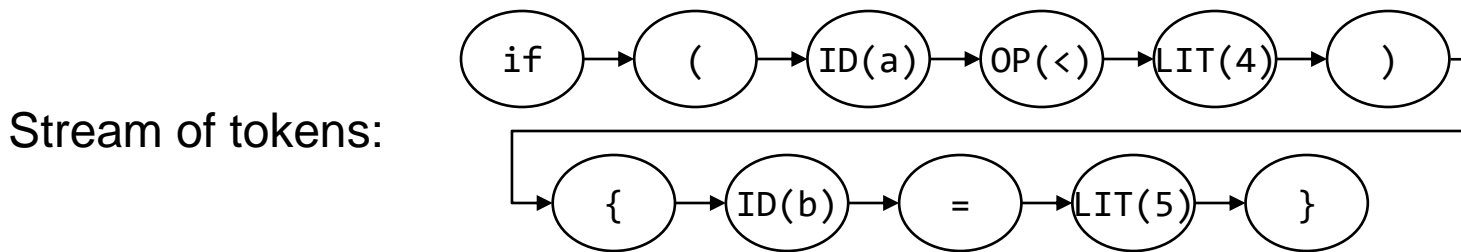
Week 4: Parsers

Parsers - Overview

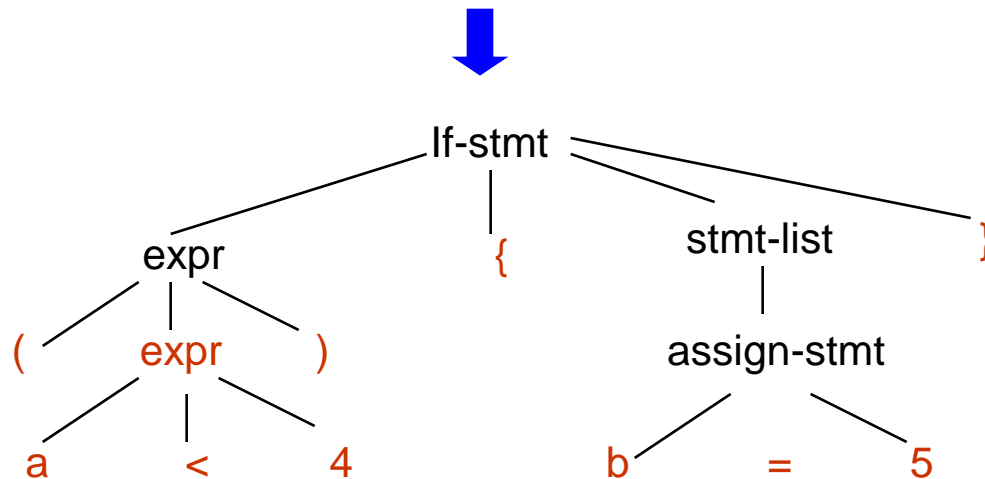
- Also called syntax analyzers
- Determine two things:
 1. If a program is valid syntactically
 - Is an English sentence grammatically correct?
 2. Structure of programming language constructs
 - E.g. the sequence `IF, ID(a), OP(<), ID(b), {, ID(a), ASSIGN, LIT(5), }, ;, }` refers to `if-statement` ?
 - Diagramming English sentences

Parsers - Overview

- Input: stream of tokens
- Output: Parse tree
 - sometimes implicit



Parse tree:



Parsers – what do we need to know?

1. How do we define language constructs?
 - Context-free grammars
2. How do we determine: 1) valid strings in the language? 2) structure of program?
 - LL Parsers, LR Parsers
3. How do we write Parsers?
 - E.g. use a parser generator tool such as [Bison](#)

Languages

- A language is (possibly infinite) set of strings
- Regular expressions describe *regular languages*
weakness: can't describe a string of the form:

$$\{ ({}^i)^i \mid i \geq 1 \}$$

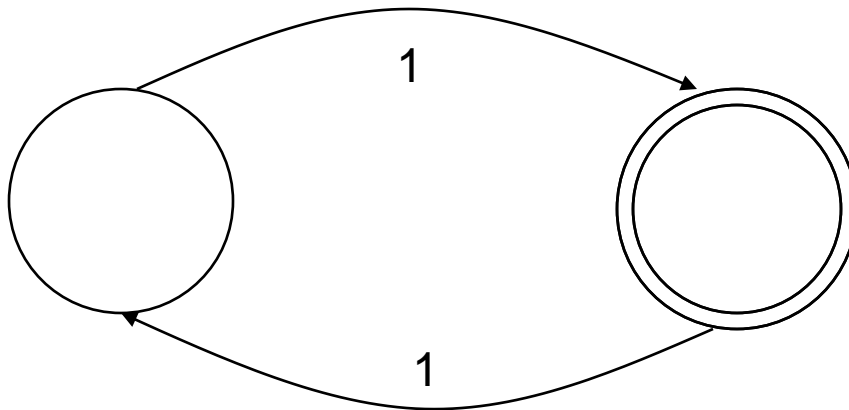
Parenthesized expressions: (((int x;)))

N **Programming language syntax is i.e. recursive**

IF
IF
FI
IF
FI

Trivia

- Regular expressions can describe strings:
 $\{ \text{mod } k \mid k = \# \text{ states in FA} \}$



“accept all strings having odd number of 1s”

Context Free Grammar (CFG)

- Natural notation for describing recursive structure definitions. Hence, suitable for specifying language constructs.
- Consist of:
 - A set of *Terminals*
 - A set of *Non-terminals*
 - A *Start Symbol*
 - A set of *Productions*

Context Free Grammar (CFG)

- Terminology:

Terminals – T

Non-terminals – N

Start Symbol – S ∈ N

Productions – P (also called rules sometimes)

$$X \longrightarrow Y_1 Y_2 Y_3 \dots Y_N \quad | \quad X \in N, Y_i \in N \cup T \cup \epsilon/\lambda$$

- Grammar $G = (T, N, S, P)$

E.g. $G = (\{a, b\}, \{S, A, B\}, S, \{S \rightarrow AB, A \rightarrow Aa, A \rightarrow a, B \rightarrow Bb, B \rightarrow b\})$

- G is context free. Why?

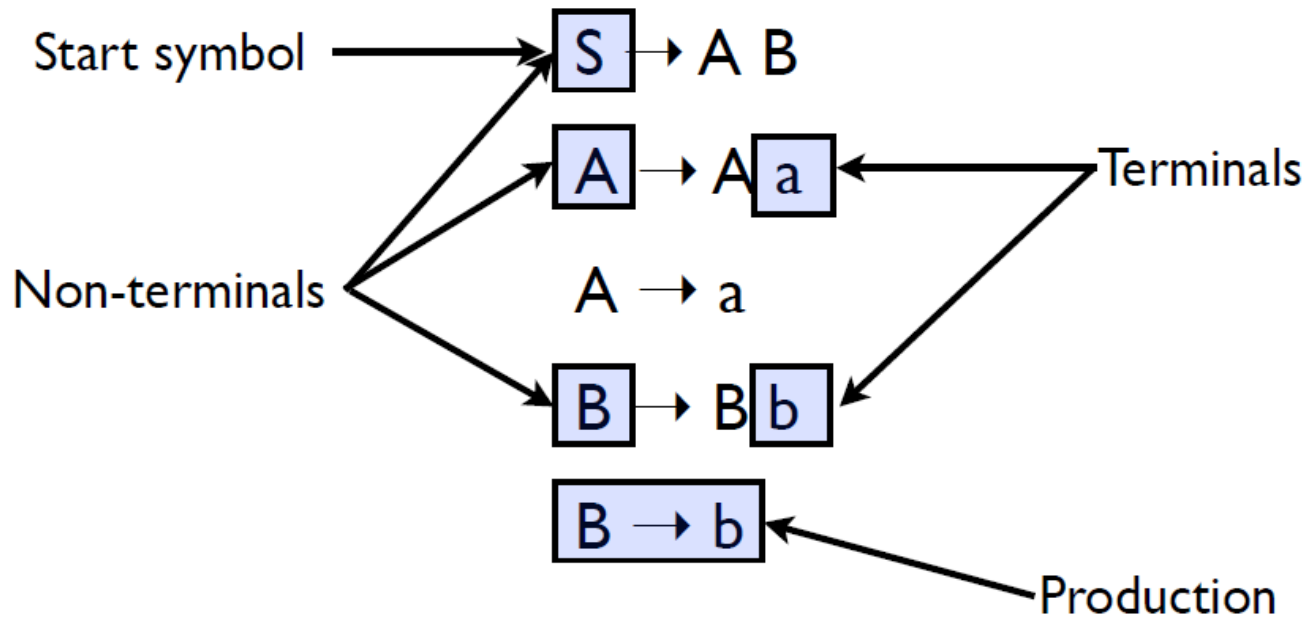
Terminology

- *Strings* are composed of symbols
 - $A A a a B b b A a$ is a string
 - We will use Greek letters to represent strings composed of both terminals and non-terminals
- $L(G)$ is the language produced by the grammar G
 - All strings consisting of only terminals that can be produced by G
 - In our example, $L(G) = a^+b^+$
 - The language of a context-free grammar is a **context-free language**
 - All regular languages are context-free, but not vice versa

String Derivations

- How do we apply the grammar rules repeatedly to determine the validity of a string? (i.e. string belongs to the language specified by the grammar)
 1. Always start with the Start Symbol
 2. Replace any Non-terminal X in the string by the right-hand side of the production
 3. Repeat Step 2 until there are no more non-terminals

Simple grammar



Backus Naur Form (BNF)

Generating strings

$S \rightarrow A B$

$A \rightarrow A a$

$A \rightarrow a$

$B \rightarrow B b$

$B \rightarrow b$

- Given a start rule, productions tell us how to rewrite a non-terminal into a different set of symbols
- Some productions may rewrite to λ . That just removes the non-terminal

To derive the string “a a b b b” we can do the following rewrites:

$S \Rightarrow A B \Rightarrow A a B \Rightarrow a a B \Rightarrow a a B b \Rightarrow$
 $a a B b b \Rightarrow a a b b b$

Exercise

Which of the below strings are accepted by the grammar:

$A \rightarrow aAa$

$A \rightarrow bBb$

$A \rightarrow \lambda$

$B \rightarrow cA$

$B \rightarrow \lambda$

1. abcba
2. abcbca
3. abba
4. abca

Programming language syntax

- Programming language syntax is defined with CFGs
- Constructs in language become non-terminals
- May use auxiliary non-terminals to make it easier to define constructs

`if_stmt` → if (`cond_expr`) then `statement` `else_part`

`else_part` → else `statement`

`else_part` → λ

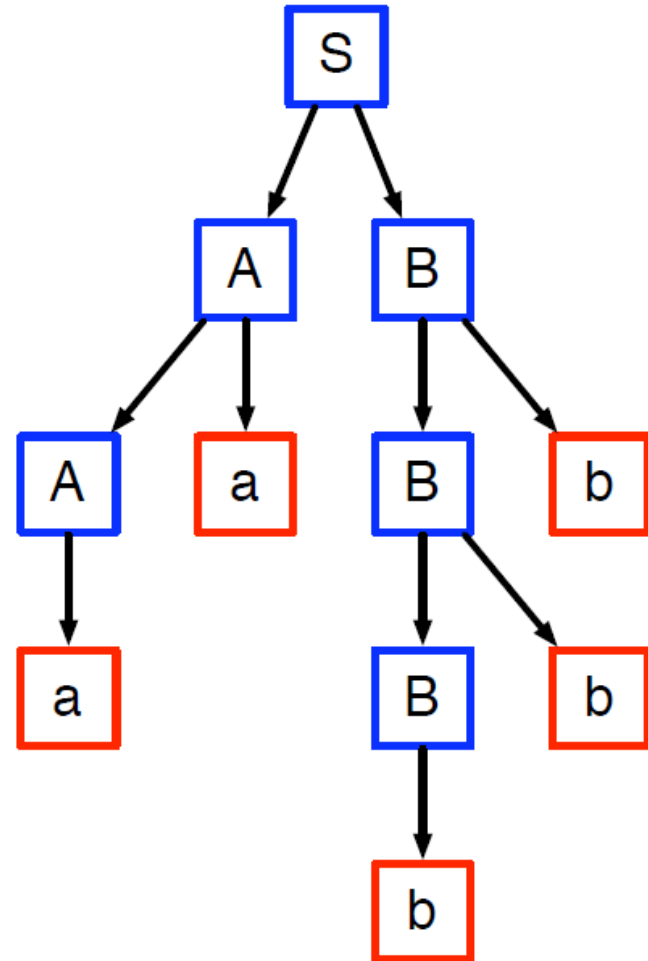
- Tokens in language become terminals

CFG Contd..

- Is it enough if parsers answer “yes” or “no” to check if a string belongs to context-free language?
 - Also need a parse tree
- What if the answer is a “no”?
 - Handle errors
- How do we implement CFGs?
 - E.g. Bison

Parse trees

- Tree which shows how a string was produced by a language
- Interior nodes of tree: non-terminals
 - Children: the terminals and non-terminals generated by applying a production rule
- Leaf nodes: terminals



Parse Trees and String Derivations

- Recall: sequence of rules applied to produce a string is a derivation
- A derivation defines a parse tree
 - A parse tree may have many derivations

Leftmost derivation

- Rewriting of a given string starts with the leftmost symbol
- Exercise: do a leftmost derivation of the input program

$F(V + V)$

using the following grammar:

E	→	Prefix (E)
E	→	V Tail
Prefix	→	F
Prefix	→	λ
Tail	→	+ E
Tail	→	λ

- What does the parse tree look like?

Rightmost derivation

- Rewrite using the rightmost non-terminal, instead of the left
- What is the rightmost derivation of this string?

$F(V + V)$

E	→	Prefix (E)
E	→	V Tail
Prefix	→	F
Prefix	→	λ
Tail	→	+ E
Tail	→	λ

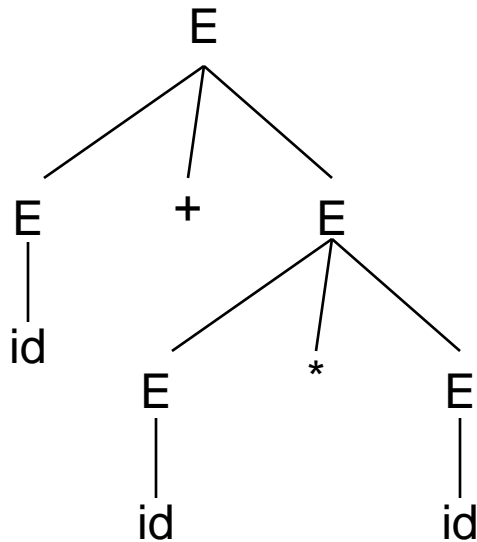
Ambiguity

- Grammar that produces more than one parse tree for some string

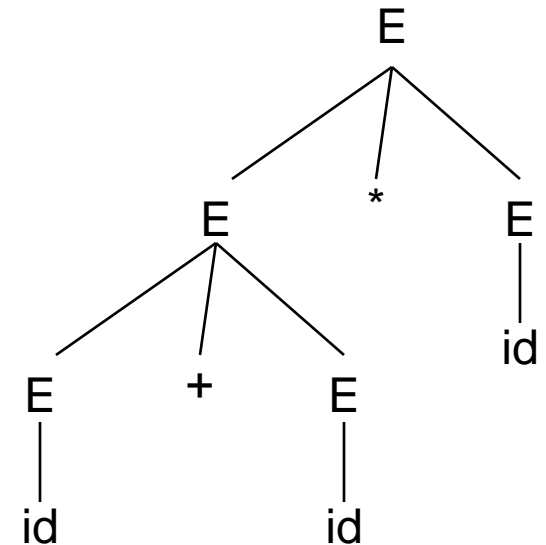
E.g. $E \rightarrow E + E \mid E * E \mid id$

String: `id+id*id`

$E \rightarrow E + E$
 $E \rightarrow id + E$
 $E \rightarrow id + E * E$
 $E \rightarrow id + id * E$
 $E \rightarrow id + id * id$



$E \rightarrow E * E$
 $E \rightarrow E + E * E$
 $E \rightarrow id + E * E$
 $E \rightarrow id + id * E$
 $E \rightarrow id + id * id$



Ambiguity – what to do?

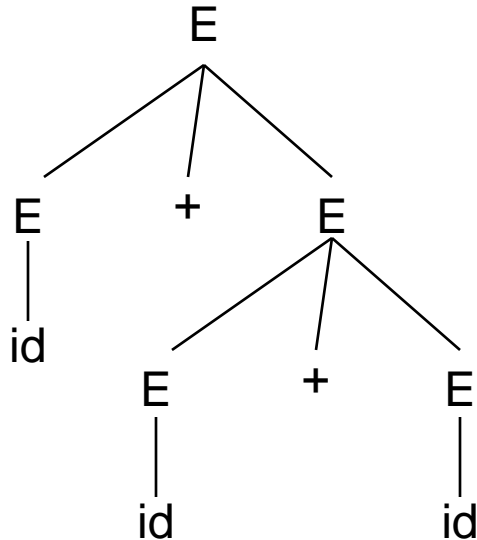
- Ignore it
 - Give hints to other components of the compiler on how to resolve it
- Fix it
 - Manually
 - May make the grammar complicated and difficult to maintain

Ambiguity – ignore

- $E \rightarrow E + E \mid id$

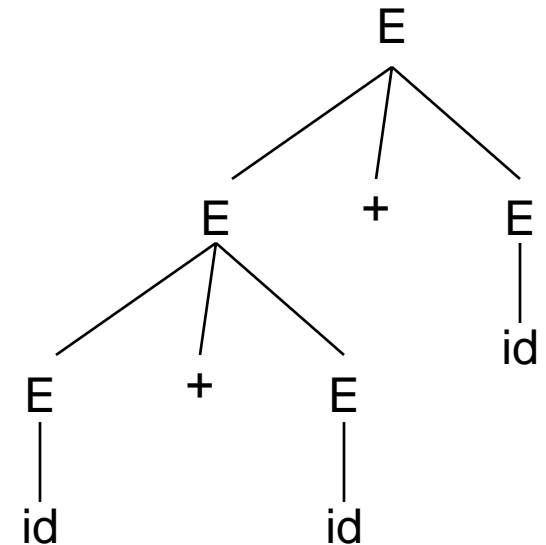
$E \rightarrow E + E$
 $E \rightarrow id + E$
 $E \rightarrow id + E + E$
 $E \rightarrow id + id + E$
 $E \rightarrow id + id + id$

Produces:
 $id + (id + id)$



$E \rightarrow E + E$
 $E \rightarrow E + E + E$
 $E \rightarrow id + E + E$
 $E \rightarrow id + id + E$
 $E \rightarrow id + id + id$

Produces:
 $(id + id) + id$



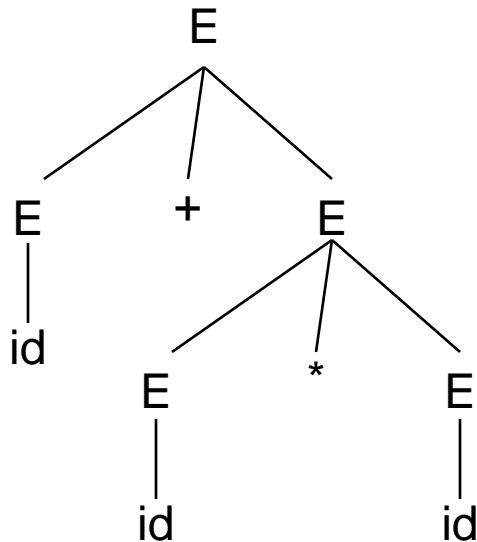
- Associativity declaration in Bison:
`%left +`

Picks the parse tree on the right

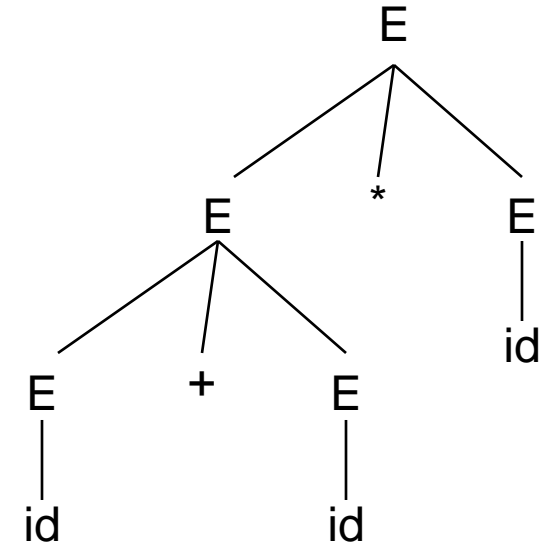
Ambiguity - ignore

- $E \rightarrow E + E \mid E * E \mid id$

$E \rightarrow E + E$
 $E \rightarrow id + E$
 $E \rightarrow id + E * E$
 $E \rightarrow id + id * E$
 $E \rightarrow id + id * id$



$E \rightarrow E * E$
 $E \rightarrow E + E * E$
 $E \rightarrow id + E * E$
 $E \rightarrow id + id * E$
 $E \rightarrow id + id * id$



%left +

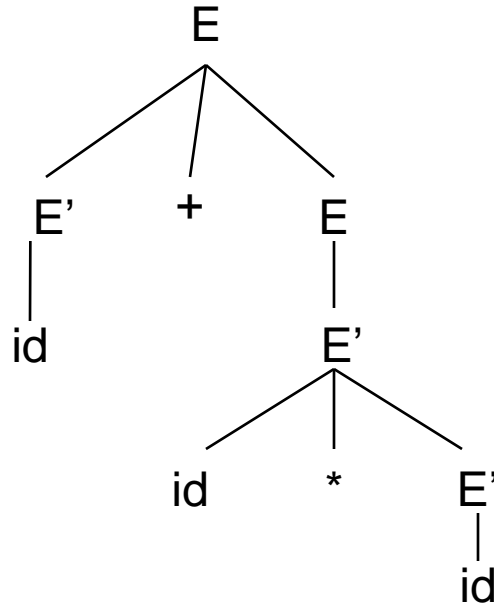
%left *

*Tells that * has higher precedence over + and both are left associative. So we get the tree on left.*

Ambiguity – fixing

- Rewrite $E \rightarrow E + E \mid E * E \mid id$ as
 $E \rightarrow E' + E \mid E'$
 $E' \rightarrow id * E' \mid id \mid (E) * E' \mid (E)$

$E \rightarrow E' + E$
 $E \rightarrow id + E$
 $E \rightarrow id + E'$
 $E \rightarrow id + id * E'$
 $E \rightarrow id + id * id$



E controls generation of $+$

E' controls generation of $*$. $*$'s are nested deeper in the parse tree.

Ambiguity - fixing

```
stmt -> if expr then stmt |  
      if expr then stmt else stmt |  
      other
```

String: if E1 then if E2 then S1 else S2

Exercise: *verify if the above grammar is ambiguous. If so, rewrite the grammar to make it unambiguous.*

```
stmt -> matched | open  
matched -> if expr then matched else matched |  
         other  
open -> if expr then stmt |  
       if expr then matched else open
```

Error Handling

- Objective: detect invalid programs and provide meaningful feedback to programmer
 - Report errors accurately
 - Recover from errors quickly
 - Don't slow down compilation

Error Types

- Many types of errors:
 - Lexical – use `Size` instead of `size`
 - Syntactic – extra brace
 - Semantic – `float sqr; sqr(2);`
 - Logical – use `=` instead of `==`

Error Handling - Types

1. Panic mode
2. Error production
3. Automatic local or global correction

Panic Mode Error Handling

- Simplest, most popular
- Discards tokens until one from a set of *synchronizing tokens* is found
- Synchronizing tokens have a clear role
e.g. semicolons, braces
- E.g. `i=i++j`
policy: while parsing an expression, discard all tokens until an integer is found. *This policy skips the additional +*
- Specifying policy in bison: **error** keyword
`E -> E + E | (E) | id | error int | error`

Error Productions

- Anticipate common errors
 - 2x instead of 2 *
- Augment the grammar
 - $E \rightarrow EE \mid \dots$
- Disadvantages:
 - Complicates the grammar

Error Corrections

- Rewrite the program – find a “nearby” correct program
 - Local corrections – insert a semicolon, replace a comma with semicolon etc.
 - Global corrections – modify the parse tree with “edit distance” metric in mind
- Disadvantages?
 - Implementation difficulty
 - Slows down compilation
 - Not sure if “nearby” program is intended

Top-down Parsing

- Also called recursive-descent parsing
- Equivalent to finding the left-derivation for an input string
 - Recall: expand the leftmost non-terminal in a parse tree
 - Expand the parse tree in pre-order i.e. identify parent nodes before children

Top-down Parsing

$S \rightarrow cAd$

$A \rightarrow ab \mid a$

String: cad

↑: next symbol to
be read

*We need to backtrack
after step 3 and reset
input pointer*

Can we do better ?

Step	Input string	Parse tree
1	cad ↑	S
2	cad ↑ ↑	<pre>graph TD S --> c S --> A S --> d A --> a A --> b</pre>
3	cad ↑	<pre>graph TD S --> c S --> A S --> d A --> a A --> b</pre>
4	cad ↑	<pre>graph TD S --> c S --> A S --> d A --> a</pre>

Top-down Parsing

- 1) $S \rightarrow F$
- 2) $S \rightarrow (S + F)$
- 3) $F \rightarrow a$

string: (a+a)

string': (a+a)\$

	()	a	+	\$
S	2	-	1	-	-
F	-	-	3	-	-

Assume that the table is given.

- Table-driven (Parse Table) approach doesn't require backtracking

But how do we construct such a table?

First and follow sets

- $\text{First}(\alpha)$: the set of terminals (and/or λ) that begin all strings that can be derived from α
 - $\text{First}(A) = \{x, y, \lambda\}$
 - $\text{First}(xA) = \{x\}$
 - $\text{First}(AB) = \{x, y, b\}$
- $\text{Follow}(A)$: the set of terminals (and/or $\$,$ but no λ s) that can appear immediately after A in some partial derivation
 - $\text{Follow}(A) = \{b\}$

$$S \rightarrow A B \$$$

$$A \rightarrow x a A$$

$$A \rightarrow y a A$$

$$A \rightarrow \lambda$$

$$B \rightarrow b$$

First and follow sets

- $\text{First}(\alpha) = \{a \in V_t \mid \alpha \Rightarrow^* a\beta\} \cup \{\lambda \mid \text{if } \alpha \Rightarrow^* \lambda\}$
- $\text{Follow}(A) = \{a \in V_t \mid S \Rightarrow^+ \dots Aa \dots\} \cup \{\$ \mid \text{if } S \Rightarrow^+ \dots A \$\}$

S: start symbol

a: a terminal symbol

A: a non-terminal symbol

α, β : a string composed of terminals and non-terminals (typically, α is the RHS of a production

\Rightarrow : derived in 1 step

\Rightarrow^* : derived in 0 or more steps

\Rightarrow^+ : derived in 1 or more steps

Computing first sets

- Terminal: $\text{First}(a) = \{a\}$
- Non-terminal: $\text{First}(A)$
 - Look at all productions for A
 $A \rightarrow X_1 X_2 \dots X_k$
 - $\text{First}(A) \supseteq (\text{First}(X_1) - \lambda)$
 - If $\lambda \in \text{First}(X_1)$, $\text{First}(A) \supseteq (\text{First}(X_2) - \lambda)$
 - If λ is in $\text{First}(X_i)$ for all i , then $\lambda \in \text{First}(A)$
- Computing $\text{First}(\alpha)$: similar procedure to computing $\text{First}(A)$

Top-down Parsing – predictive parsers

- Idea: we know sentence has to start with initial symbol
- Build up partial derivations by *predicting* what rules are used to expand non-terminals
 - Often called *predictive parsers*
- If partial derivation has terminal characters, *match* them from the input stream

A simple example

$S \rightarrow A B c \$$

$A \rightarrow x a A$

$A \rightarrow y a A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

$x a c c \$$

A simple example

$S \rightarrow A B c \$$

$A \rightarrow x a A$

$A \rightarrow y a A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

$x a c c \$$

special "end of input" symbol

A simple example

$S \rightarrow A B c \$$

$A \rightarrow x a A$

$A \rightarrow y a A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

• A sentence in the grammar:

$x a c c \$$

Current derivation: S

A simple example

$S \rightarrow A B c \$$

$A \rightarrow x a A$

$A \rightarrow y a A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

$x a c c \$$

Current derivation: $A B c \$$

Predict rule

A simple example

$S \rightarrow A B c \$$

Choose based on
first set of rules

$A \rightarrow x a A$
 $A \rightarrow y a A$
 $A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

• A sentence in the grammar:

$x a c c \$$

Current derivation: $x a A B c \$$

Predict rule *based on next token*

A simple example

$S \rightarrow A B c \$$

$A \rightarrow x a A$

$A \rightarrow y a A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

$x a c c \$$

Current derivation: $x a A B c \$$

Match token

A simple example

$S \rightarrow A B c \$$

$A \rightarrow x a A$

$A \rightarrow y a A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

• A sentence in the grammar:

$x a c c \$$

Current derivation: $x a A B c \$$

Match token

A simple example

$S \rightarrow A B c \$$

Choose based on
first set of rules

$A \rightarrow x a A$
 $A \rightarrow y a A$
 $A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

• A sentence in the grammar:

$x a c c \$$

Current derivation: $x a c B c \$$

Predict rule *based on next token*

A simple example

$S \rightarrow A B c \$$

$A \rightarrow x a A$

$A \rightarrow y a A$

$A \rightarrow c$

$B \rightarrow b$

• A sentence in the grammar:

$B \rightarrow \lambda$

$x a c c \$$

Current derivation: $x a c B c \$$

Match token

A simple example

$S \rightarrow A B c \$$

$A \rightarrow x a A$

Choose based on
follow set

$A \rightarrow y a A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

• A sentence in the grammar:

$x a c c \$$

Current derivation: $x a c \lambda c \$$

Predict rule *based on next token*

A simple example

$S \rightarrow A B c \$$

$A \rightarrow x a A$

$A \rightarrow y a A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

• A sentence in the grammar:

$x a c c \$$

Current derivation: $x a c c \$$

Match token

A simple example

$S \rightarrow A B c \$$

$A \rightarrow x a A$

$A \rightarrow y a A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

$x a c c \$$

Current derivation: $x a c c \$$

Match token

Top-down vs. Bottom-up parsers

- Top-down parsers expand the parse tree in *pre-order*
 - Identify parent nodes before the children
- Bottom-up parsers expand the parse tree in *post-order*
 - Identify children before the parents
- Notation:
 - LL(1): Top-down derivation with 1 symbol lookahead
 - LL(k): Top-down derivation with k symbols lookahead
 - LR(1): Bottom-up derivation with 1 symbol lookahead

Suggested Reading

- Alfred V. Aho, Monica S. Lam, Ravi Sethi and Jeffrey D. Ullman: Compilers: Principles, Techniques, and Tools, 2/E, AddisonWesley 2007
 - Chapter 4 (Sections: 4.1 to 4.4)
- Fisher and LeBlanc: Crafting a Compiler with C
 - Chapter 4, Chapter 5 (Sections 5.1 to 5.5, 5.9)