

Dataflow Analysis

Week 13: Constant Propagation

(Slides courtesy: Prof. Milind Kulkarni)

Program Optimizations

- So far we have talked about different kinds of optimizations
 - Peephole optimizations
 - Local common sub-expression elimination
- What about *global optimizations*
 - Optimizations across multiple basic blocks (usually a whole procedure)
 - Not just a single loop

Useful optimizations

- Common subexpression elimination (global)
 - Need to know which expressions are available at a point
- Dead code elimination
 - Need to know if the effects of a piece of code are never needed, or if code cannot be reached
- Constant folding
 - Need to know if variable has a constant value
- So how do we get this information?

Dataflow analysis

- Framework for doing compiler analyses to drive optimization
- Works across basic blocks
- Examples
 - Constant propagation: determine which variables are constant
 - Liveness analysis: determine which variables are live
 - Available expressions: determine which expressions are have valid computed values
 - Reaching definitions: determine which definitions could “reach” a use



Example: Constant Propagation and Dead Code Elimination

X = 1
Y = X + 2
Z = Y + A



X = 1
Y = 1 + 2
Z = Y + A



~~X = 1~~
Y = 1 + 2
Z = Y + A

Constant Propagation

Dead Code Elimination

Example: constant propagation

- Goal: determine when variables take on constant values
- Why? Can enable many optimizations
 - Constant folding

```
x = 1;  
y = x + 2;  
if (x > z) then y = 5  
... y ...
```



```
x = 1;  
y = 3;  
if (x > z) then y = 5  
... y ...
```

- Create dead code

```
x = 1;  
y = x + 2;  
if (y > x) then y = 5  
... y ...
```



```
x = 1;  
y = 3; //dead code  
if (true) then y = 5 //simplify!  
... y ...
```



Exercise – Constant Propagation

1. `X := 2`
2. **Label1:**
3. `Y := X + 1`
4. `if Z > 8 goto Label2`
5. `X := 3`
6. `X := X + 5`
7. `Y := X + 5`
8. `X := 2`
9. `if Z > 10 goto Label1`
10. `X := 3`
11. **Label2:**
12. `Y := X + 2`
13. `X := 0`
14. `goto Label3`
15. `X := 10`
16. `X := X + X`
17. **Label3:**
18. `Y := X + 1`

Which lines using X could be replaced with a constant value? (apply only constant propagation)

How can we find constants?

- Ideal: run program and see which variables are constant
 - Problem: variables can be constant with some inputs, not others – need an approach that works for all inputs!
 - Problem: program can run forever (infinite loops?) – need an approach that we know will finish
- Idea: run program *symbolically*
 - Essentially, keep track of whether a variable is constant or not constant (but nothing else)

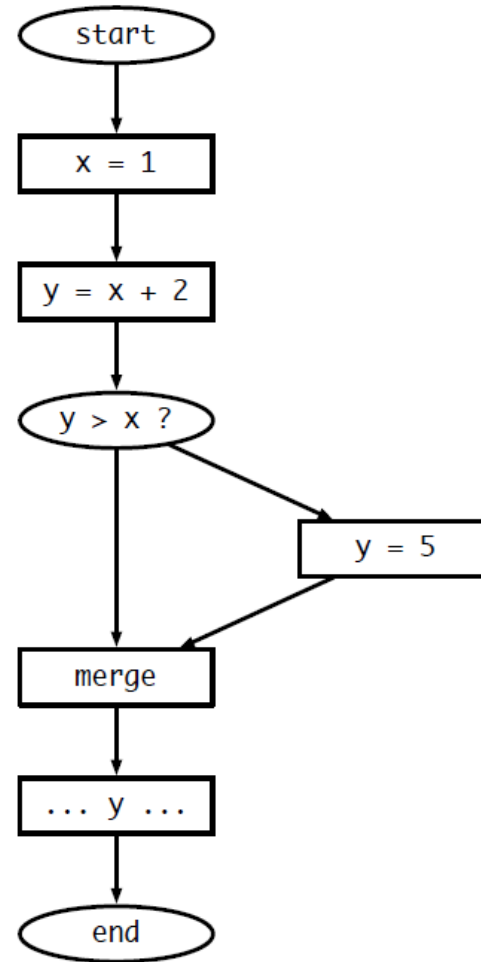
Overview of algorithm

- Build control flow graph
 - We'll use statement-level CFG (with merge nodes) for this
- Perform symbolic evaluation
 - Keep track of whether variables are constant or not
- Replace constant-valued variable uses with their values, try to simplify expressions and control flow



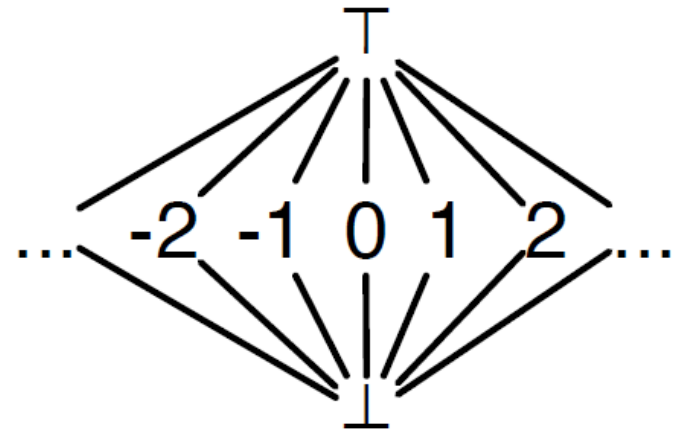
Build CFG

```
x = 1;  
y = x + 2;  
if (y > x) then y = 5;  
... y ...
```



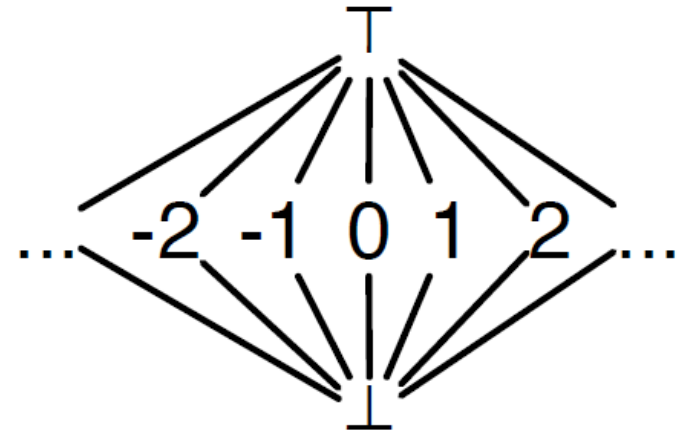
Symbolic evaluation

- Idea: replace each value with a symbol
- constant (specify which), no information, definitely not constant
- Can organize these possible values in a *lattice*
- Set of possible values, arranged from least information to most information



Symbolic evaluation

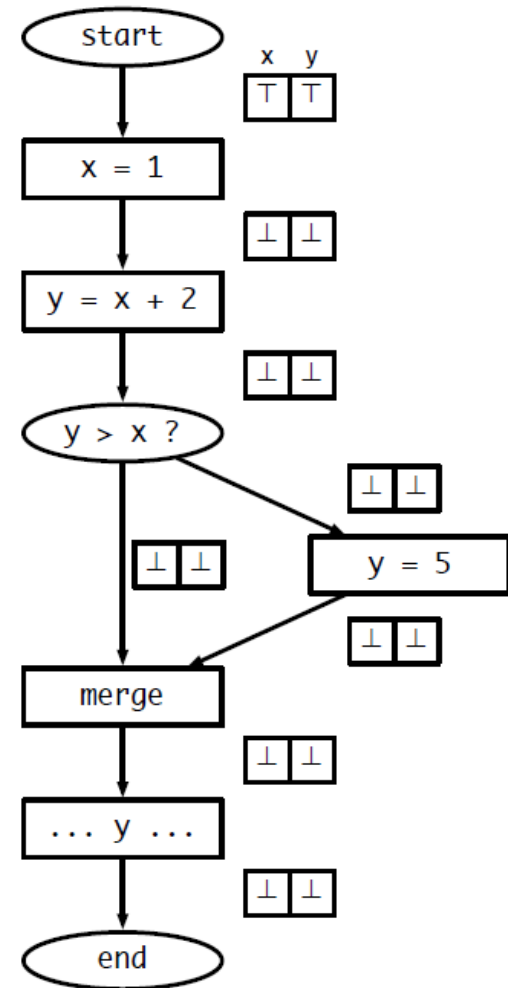
- Evaluate expressions symbolically:
 $\text{eval}(e, V_{\text{in}})$
- If e evaluates to a constant, return that value. If any input is \top (or \perp), return \top (or \perp)
 - Why?
- Two special operations on lattice
 - $\text{meet}(a, b)$ – highest value less than or equal to both a and b
 - $\text{join}(a, b)$ – lowest value greater than or equal to both a and b



Join often written as $a \sqcup b$
Meet often written as $a \sqcap b$

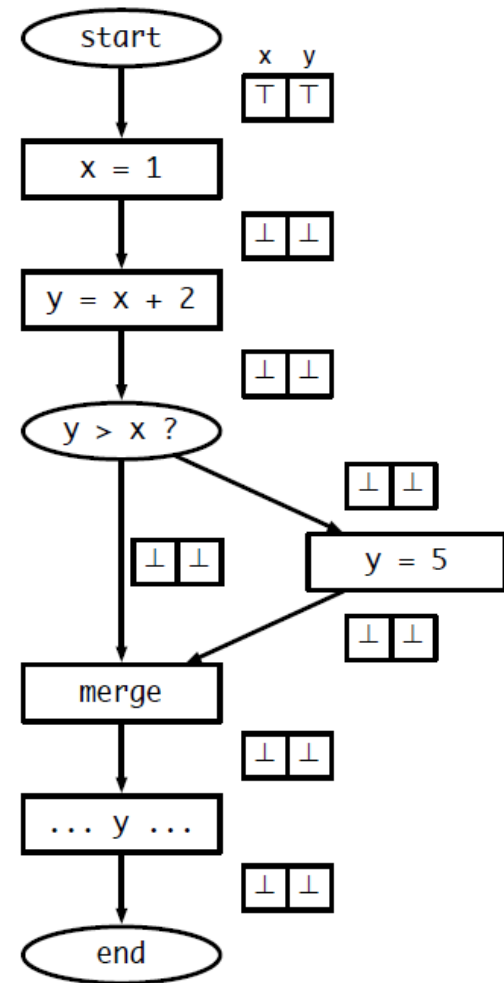
Putting it together

- Keep track of the symbolic value of a variable at every program point (on every CFG edge)
- State vector
- What should our initial value be?
- Starting state vector is all \top
 - Can't make any assumptions about inputs – must assume not constant
- Everything else starts as \perp , since we have no information about the variable at that point



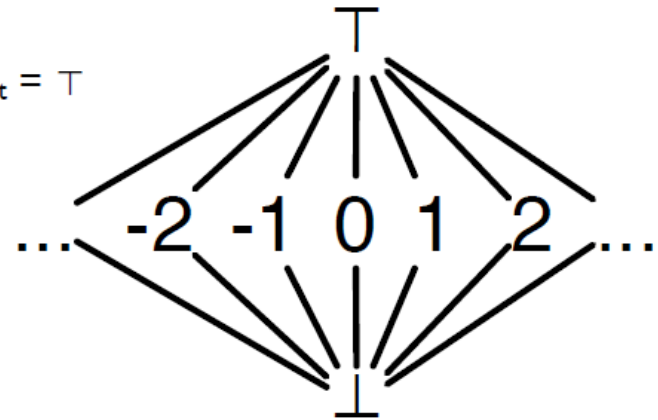
Executing symbolically

- For each statement $t = e$ evaluate e using V_{in} , update value for t and propagate state vector to next statement
- What about switches?
 - If e is true or false, propagate V_{in} to appropriate branch
 - What if we can't tell?
 - Propagate V_{in} to both branches, and symbolically execute both sides
- What do we do at merges?



Handling merges

- Have two different V_{in} s coming from two different paths
- Goal: want new value for V_{in} to be *safe* (shouldn't generate wrong information), and we don't know which path we actually took
- Consider a single variable. Several situations:
 - $V_1 = \perp, V_2 = * \rightarrow V_{out} = *$
 - $V_1 = \text{constant } x, V_2 = x \rightarrow V_{out} = x$
 - $V_1 = \text{constant } x, V_2 = \text{constant } y \rightarrow V_{out} = \top$
 - $V_1 = \top, V_2 = * \rightarrow V_{out} = \top$
- Generalization:
 - $V_{out} = V_1 \sqcup V_2$





Result: worklist algorithm

- Associate state vector with each edge of CFG, initialize all values to \perp , worklist has just start edge

- While worklist not empty, do:

Process the next edge from worklist

Symbolically evaluate target node of edge using input state vector

If target node is assignment ($x = e$), propagate $V_{in}[eval(e)/x]$ to output edge

If target node is branch ($e?$)

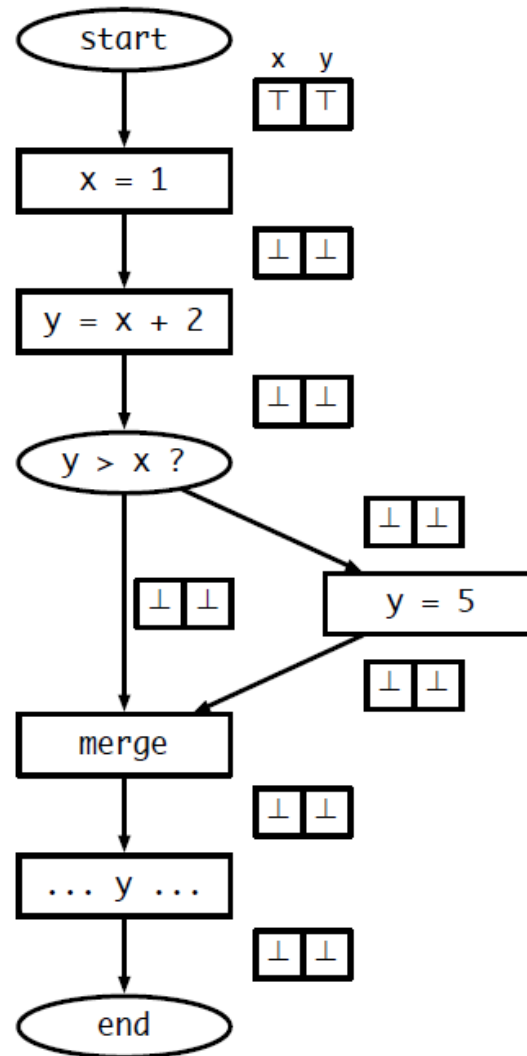
If $eval(e)$ is true or false, propagate V_{in} to appropriate output edge

Else, propagate V_{in} along both output edges

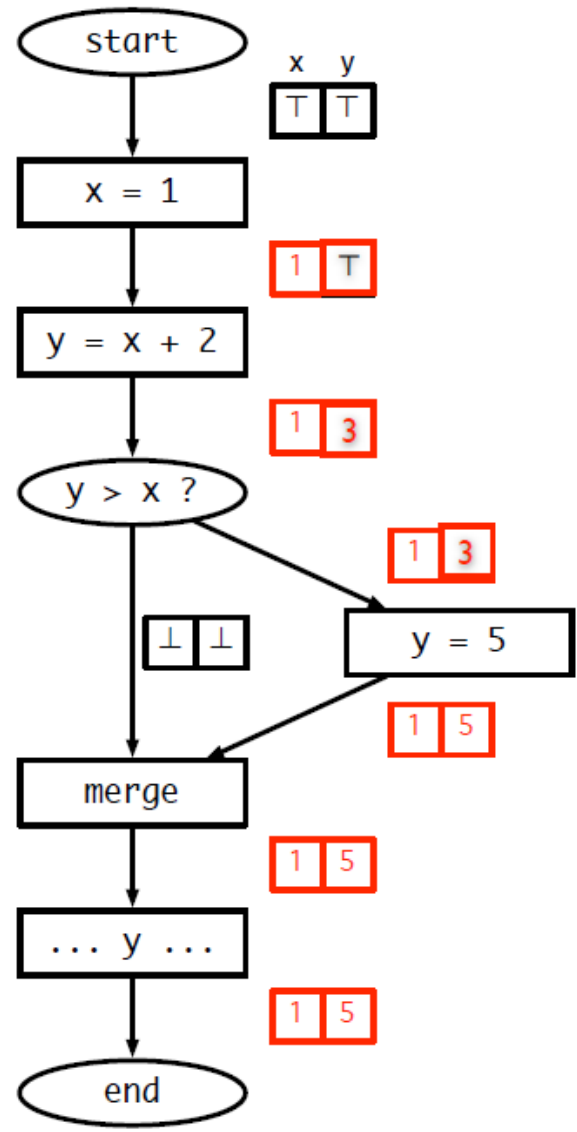
If target node is merge, propagate $join(all V_{in})$ to output edge

If any output edge state vector has changed, add it to worklist

Running example



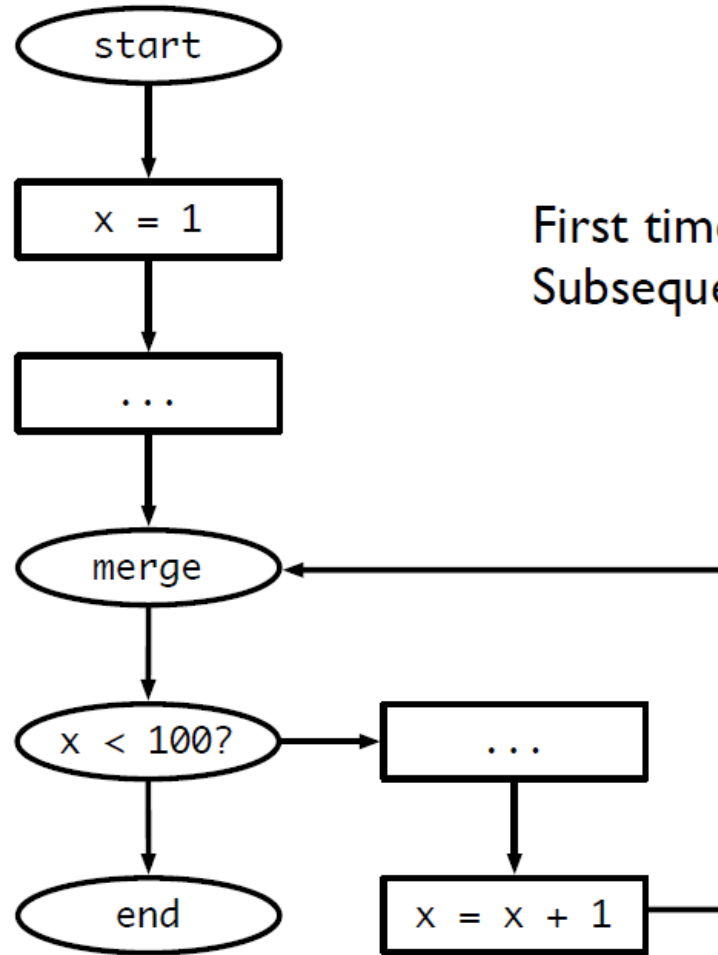
Running example



What do we do about loops?

- Unless a loop never executes, symbolic execution looks like it will keep going around to the same nodes over and over again
- Insight: if the input state vector(s) for a node don't change, then its output doesn't change
 - If input stops changing, then we are done!
- Claim: input will eventually stop changing. Why?

Loop example



First time through loop, $x = 1$
Subsequent times, $x = T$

Complexity of algorithm

- $V = \#$ of variables, $E = \#$ of edges
- Height of lattice = 2 \rightarrow each state vector can be updated at most $2 * V$ times.
- So each edge is processed at most $2 * V$ times, so we process at most $2 * E * V$ elements in the worklist.
- Cost to process a node: $O(V)$
- Overall, algorithm takes $O(EV^2)$ time

Question

- Can we generalize this algorithm and use it for more analyses?

Constant propagation

- Step 1: choose lattice (which values are you going to track during symbolic execution)?
 - Use constant lattice
- Step 2: choose direction of dataflow (if executing symbolically, can run program backwards!)
 - Run forward through program
- Step 3: create *transfer functions*
 - How does executing a statement change the symbolic state?
- Step 4: choose *confluence operator*
 - What do do at merges? For constant propagation, use join