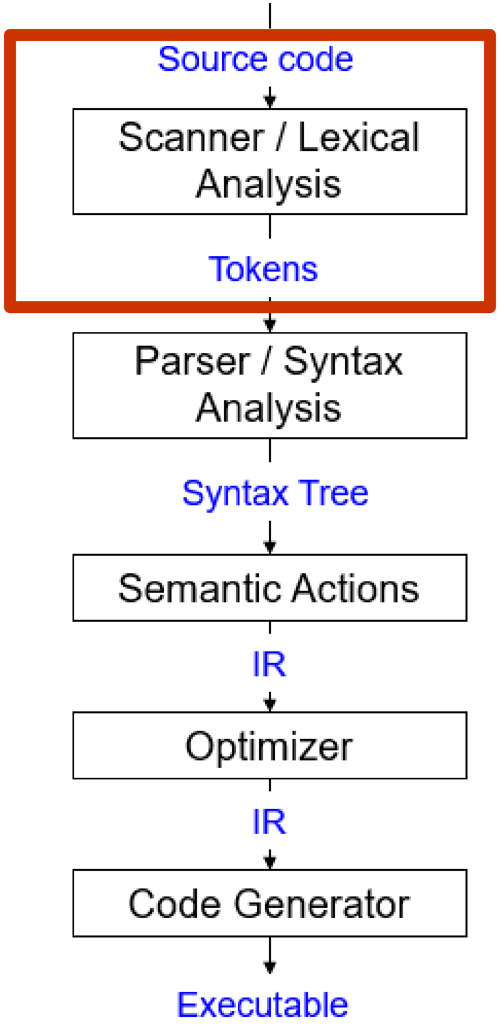
CS323: Compilers

Spring 2023

Week 3: Scanners (conclusion), Parsers

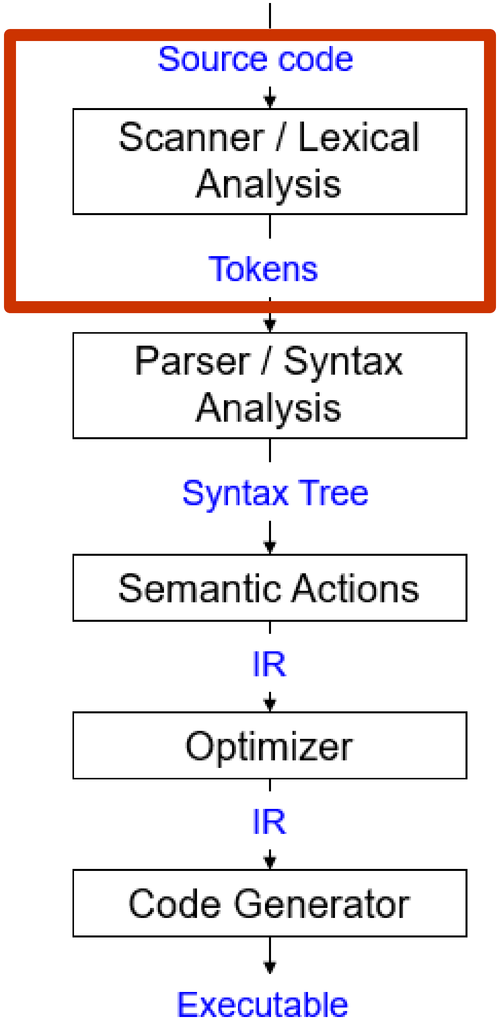
Scanners (Summary)

- Also called *Lexers* / *Lexical Analyzers*
- Input: stream of letters (program text / source code), Output: sequence / list of *tokens*
- Token: a pair <category/class, value>
  - Category defines a string *pattern*
  - Value also called *lexeme*
  - Value is a *prefix* (and hence, is a substring)
  - Value matches on of the patterns that category defines
- Scan *left-to-right* in program text, *look-ahead* to identify tokens.
  - Look-ahead buffer size determined by language design



Scanners (Summary)

- *Regular expressions* are used to formally define the patterns specified by token classes.
  - Some customization done while defining regular expressions: 1) Match the longest substring possible 2) Handle errors
- Tools such as Flex and ANTLR convert regular expressions to code. The code is your scanner implementation
  - The implementation typically converts regular expressions to *Finite Automata* (special kind of state diagram)
    - Automata are coded using efficient algorithms (E.g. Table-lookup method)
  - Efficient algorithms exist for substring matching (requiring single-pass over input program text)
    - Aho-Corasic, Knuth-Morris-Pratt (KMP)



Parsers - Overview

- Also called syntax analyzers
- Determine two things:
  1. Is a program syntactically valid?

     (Analogy) is an English language sentence grammatically correct?
  2. What is the structure of programming language constructs? E.g. does the sequence*

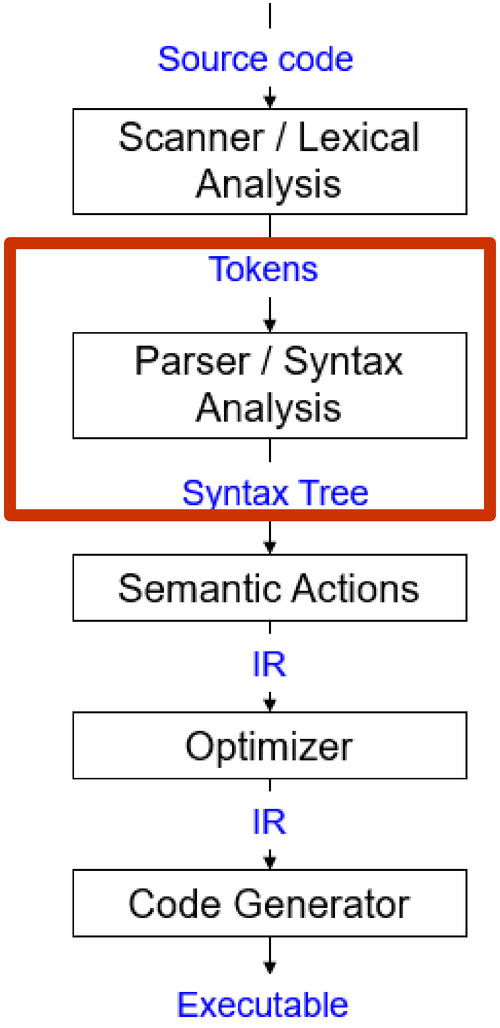
`IF, ID(a), OP(<), ID(b), {, ID(a), ASSIGN, LIT(5), }}`

refer to an `if` statement?

(Analogy) diagramming English sentences

* Correponding program text:

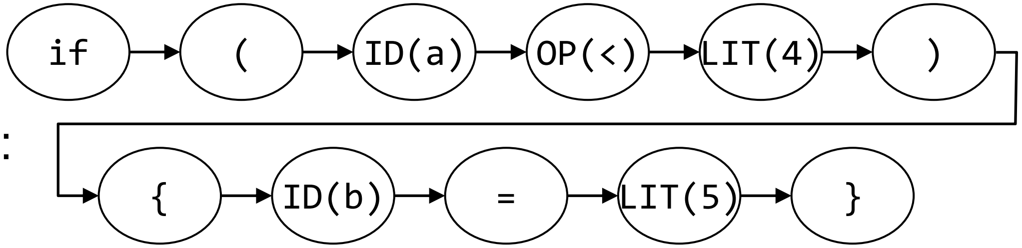
```
if (a < 4) {
    b = 5
}
```



Parsers - Overview

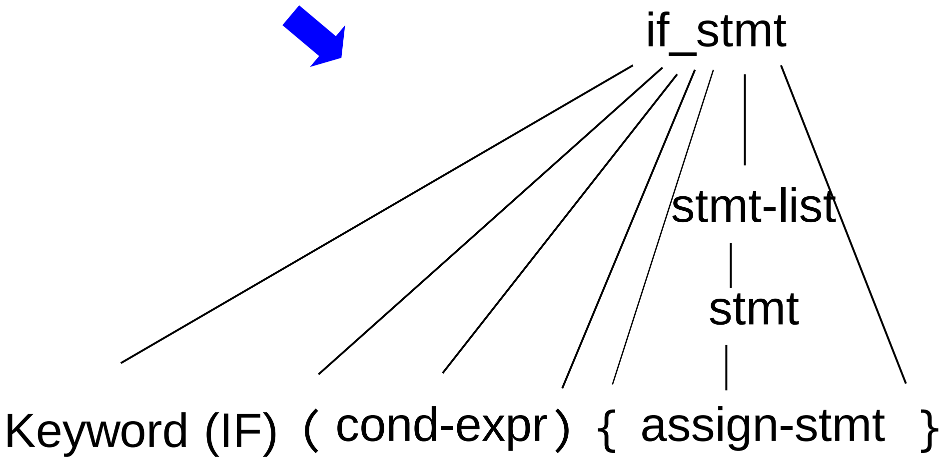
- Input: stream of tokens
- Output: Parse tree
    - sometimes implicit

Stream of tokens:



if → (→ ID(a) → OP(<) → LIT(4) →) → { → ID(b) → = → LIT(5) → }

Parse tree:



if_stmt

stmt-list

stmt

Keyword (IF) (cond-expr) { assign-stmt }

Parsers – what do we need to know?

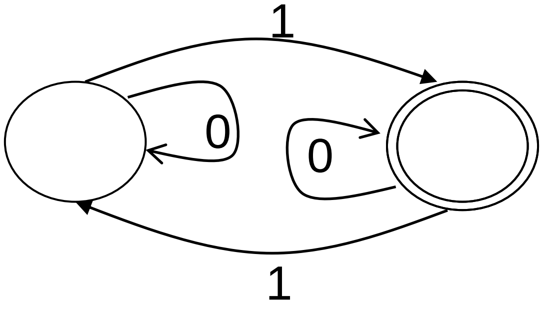
1. How do we define language constructs?
    - Context-free grammars
2. How do we determine: 1) valid strings in the language? 2) structure of program?
    - LL Parsers, LR Parsers
3. How do we write Parsers?
    - E.g. use a parser generator tool such as Bison

Languages

- A language is (possibly infinite) set of strings
- Regular expressions specify *regular languages.* However, regular languages are *weak formal languages* to describe the features of a practical programming language.

What set of strings does this FA accept?

The FA shown accepts all string with *odd number of 1s.*



What is the regular expression for the FA?

(0*10*)(10*10*)*

Regular expressions can describe strings specifying *parity*:

`{ mod k | k=# states in FA}`

weakness: regular expressions can't describe a string of the form: $\{ (^i\ )^i \mid i>=1\}$

Regular Languages

- Regular expressions can't describe a string of the form:

$$\{ (^{i})^{i} \mid i>=1\}$$

E.g. Parenthesized expressions

```
((2+3)*5)
```

Programming language syntax is i.e. recursive

```
((( int x; )))
```

Nested structures:

```
IF
  IF
    IF
    FI
  FI
FI
```

Context Free Grammar (CFG)

- Natural notation for describing recursive structure definitions. Hence, suitable for specifying language constructs.
- Consist of:
  - A set of *Terminals* (T)
  - A set of *Non-terminals* (N)
  - A *Start Symbol* ($S \in N$)
  - A *set of Productions* ($X \rightarrow Y_1 .. Y_N$) (aka. rules)

$P: X \longrightarrow Y_1 Y_2 Y_3 .. Y_N \qquad X \in N, \qquad Y_i \in N \cup T \cup \epsilon / \lambda$

Context Free Grammar (CFG)

- Grammar `G = (T, N, S, P)`

  `E.g. G = ({a,b}, {S, A, B}, S, {S → AB, A → Aa A → a, B → Bb, B → b})`

- Implicit meanings
  - First rule listed in the set of productions contains start symbol (on the left-hand side)
  - In the set of productions, you can replace the symbol X (appearing on the right-hand side only) with the string of symbols that are on the right-hand side of a rule, which has X (on the left-hand side)

Context Free Grammar (CFG)

1. Begin with only S as the initial string
2. Replace S
   - S replaced with AB
3. Repeat 2 until the string contains only terminals
   i. AB replaced with aB
   ii. aB replaced with ab

```
G = (T, N, S, P)
P:{ S->AB,
   A->Aa,
   A->a,
   B->Bb,
   B->b }
```

Summary: we move from S to a string of terminals through a series of transformations:

α_0-> … -> α_n where $\alpha_1 \, . \, . \, . \, \alpha_n$ are strings

Shorthand notation: $\alpha_0 \overset{*}{->} \alpha_n$

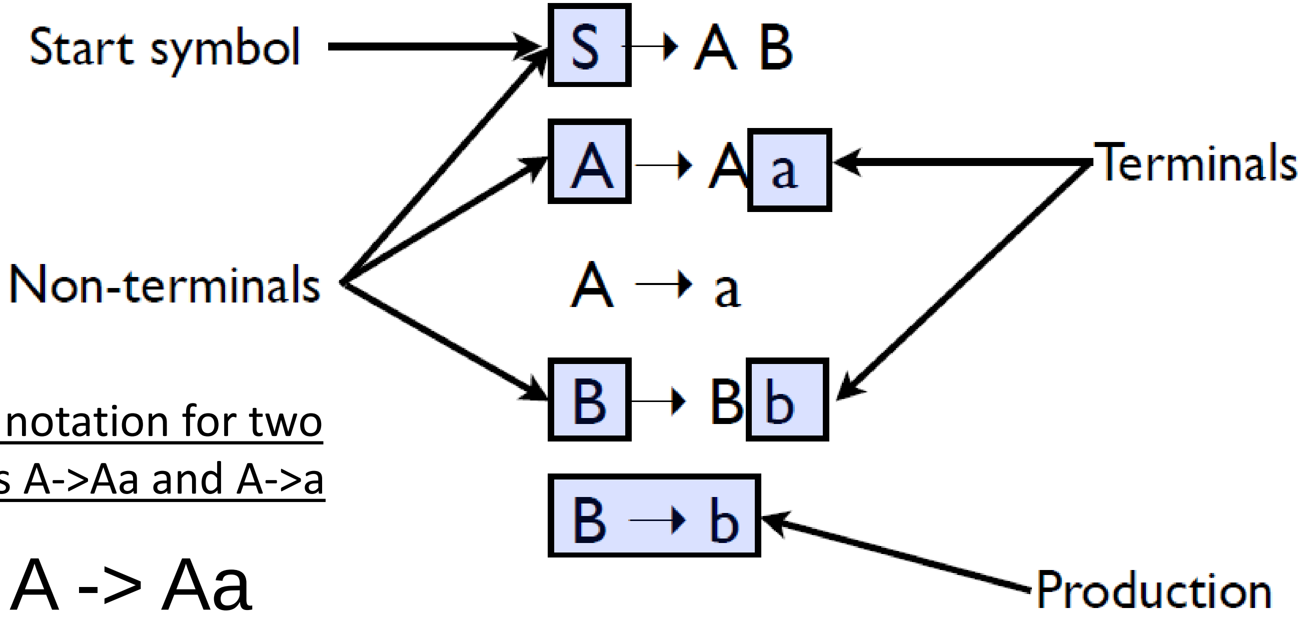
Detour: Context-Sensitive Grammar

- Can have context-sensitive grammar and languages (think: aB->ab)
  - Cannot replace right-hand side with left-hand side irrespective of the context.
  - E.g. aB->ab lays down a context: ‘a’ must be a prefix in order to transform the string “aB” to a string of terminals “ab”
    - ccaBb can be replaced by ccabb

Is grammar G context-free?

```
G = (T, N, S, P)
P:{ S->AB,
   A->Aa,
   A->a,
   B->Bb,
   B->b }
```

Simple grammar (Summary)



Start symbol

Non-terminals

Terminals

Production

S → A B

A → A a

A → a

B → B b

B → b

<u>Alternative notation for two productions A->Aa and A->a</u>

A -> Aa
| a

Backus Naur Form (BNF)

Slide courtesy: Milind Kulkarni

Programming language syntax

- Programming language syntax is defined with CFGs
- Constructs in language become non-terminals
  - May use auxiliary non-terminals to make it easier to define constructs

if_stmt → if (cond_expr) then statement else_part

else_part → else statement

else_part → λ

- Tokens in language become terminals

Slide courtesy: Milind Kulkarni

Language of the Grammar

- Language L(G) of the context-free grammar G
  - Set of strings that can be derived from S
  - $\{a_1 a_2 a_3 .. a_N \mid a_i \in T \ \forall i \text{ and } S \overset{*}{->} a_1 a_2 a_3 .. a_N\}$
  - Is called context-free language
    - All regular languages are context-free but not vice-versa.
    - Can have many grammars generating same language.

String Derivations: Does a string belong to the Language?

- How do we apply the grammar rules to determine the acceptability of a string? (i.e. the string belongs to the language, *L(G),* specified by the CFG *G*)
  - Begin with S
  - Replace S
  - Repeat till string contains terminals only. Why terminals only?

L(G) must contain strings of terminals only

- Notation:
  - We will use Greek letters to denote strings containing non-terminals and terminals
- Derivations: sequence of rules applied to produce the string of terminals

Generating strings (Example)

$S \rightarrow A\ B$

$A \rightarrow A\ a$

$A \rightarrow a$

$B \rightarrow B\ b$

$B \rightarrow b$

- Given a start rule, productions tell us how to rewrite a non-terminal into a different set of symbols
- Some productions may rewrite to λ. That just removes the non-terminal

To derive the string "a a b b b" we can do the following rewrites:

$$S \Rightarrow A\ B \Rightarrow A\ a\ B \Rightarrow a\ a\ B \Rightarrow a\ a\ B\ b \Rightarrow$$
$$a\ a\ B\ b\ b \Rightarrow a\ a\ b\ b\ b$$

Slide courtesy: Milind Kulkarni

CFG and Parsers

- Is it enough if parsers answer “yes” or “no” to check if a string belongs to context-free language?
  - Also need a parse tree
- What if the answer is a “no”?
  - Handle errors
- How do we implement CFGs?
  - E.g. Bison

Exercise

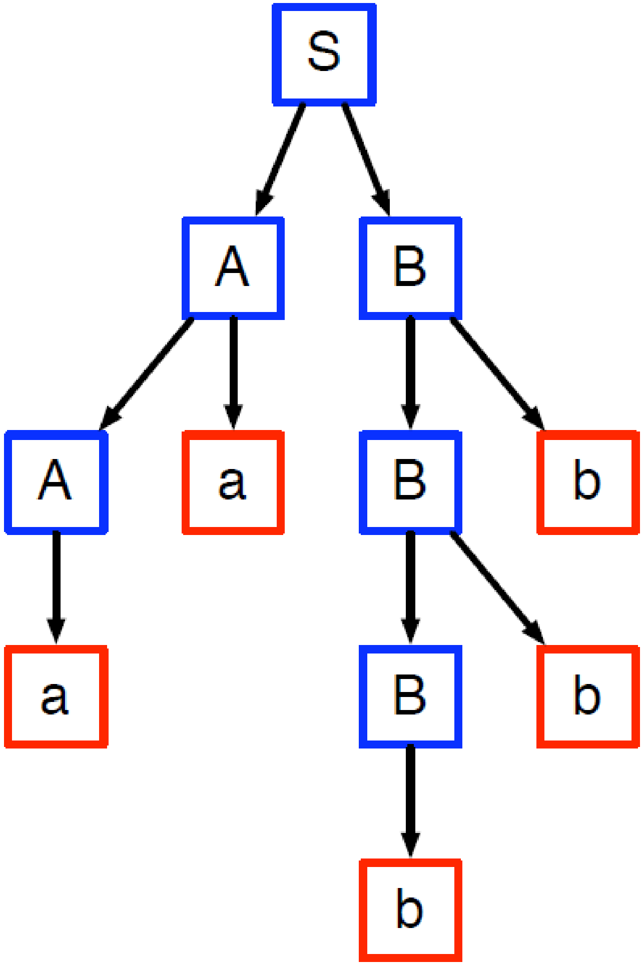
Which of the below strings are accepted by the grammar:

```
1: A ->  aAa
2: A ->  bBb
3: A ->  λ
4: B ->  cA
5: B ->  λ
```

1. abcba 1->2->4->3
2. abcbca
3. abba 1->2->5
4. abca

Parse trees

- Tree which shows how a string was produced by a language
  - Interior nodes of tree: non-terminals
    - Children: the terminals and non-terminals generated by applying a production rule
  - Leaf nodes: terminals



Slide courtesy: Milind Kulkarni

Derivations and Parse Trees

- Recall: Derivation is a sequence of rules applied to produce a string
  - $S \rightarrow \alpha_0 \rightarrow \alpha_1 \rightarrow \alpha_2 \rightarrow . . . \rightarrow \alpha_n$
- A derivation defines a parse tree
  - Parse tree is an alternative way to gather information on how the string was derived
  - A parse tree may have many derivations (think: different permutations of α)

Derivations and Parse Trees

- Consider the grammar with the following rules:

```
1: E -> E + E
2:      | E * E
3:      | id
```

- Produce derivations for the string: **`id*id+id`**

Derivations and Parse Trees

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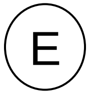
`Apply 1:` `Start with E,` **`the start symbol`**   **`Parse Tree`**

`E`



Derivations and Parse Trees

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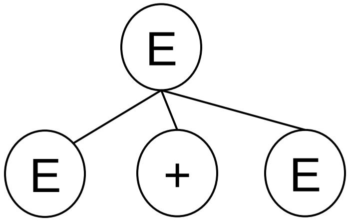
- Produce derivations for the string: **`id*id+id`**

`Apply 1:` **Replace E with E + E**

E
E+E

`Parse Tree`



Derivations and Parse Trees

- Consider the grammar with the following rules:

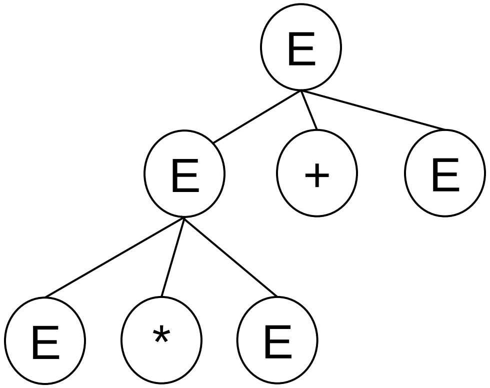
```
1: E -> E + E
2:      | E * E
3:      | id
```

- Produce derivations for the string: **`id*id+id`**

`Apply 2:` **Replace E with E * E**

`Parse Tree`

E
E+E
E*E+E



Derivations and Parse Trees

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1: E -> E + E
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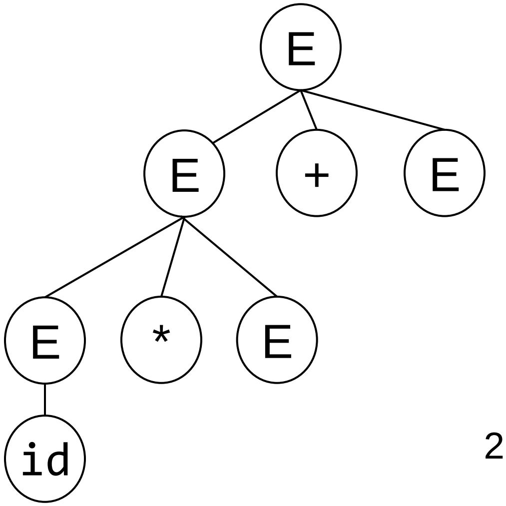
```

- Produce derivations for the string: **`id*id+id`**

`Apply 3:` **Replace E with id**

`Parse Tree`

```
E
E+E
E*E+E
id*E+E
```



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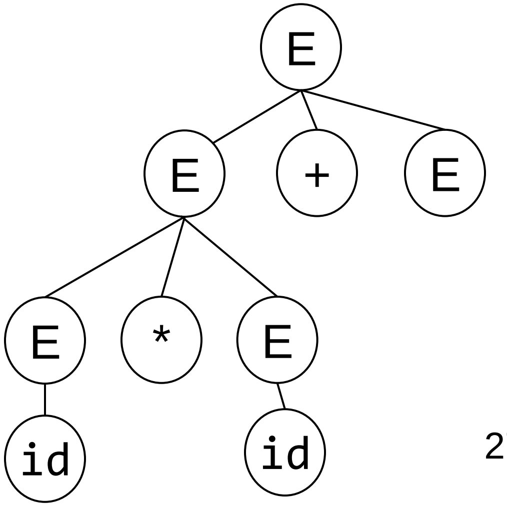
Apply 3: Replace E with id

Parse Tree

E
E+E
E*E+E
id*E+E
id*id+E



Derivations and Parse Trees

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Apply 3: **Replace E with id**

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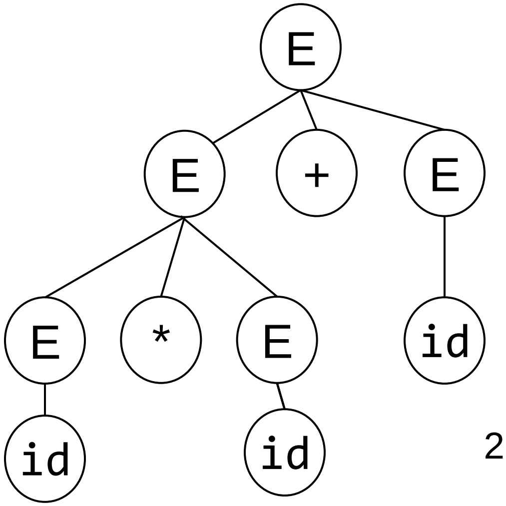
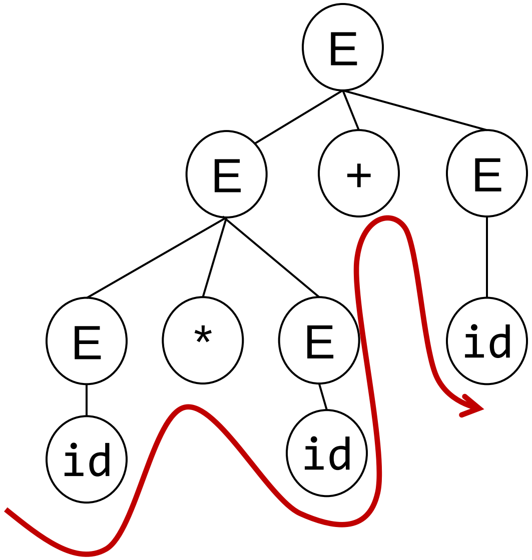
id*id+id

Parse Tree



Derivations and Parse Trees

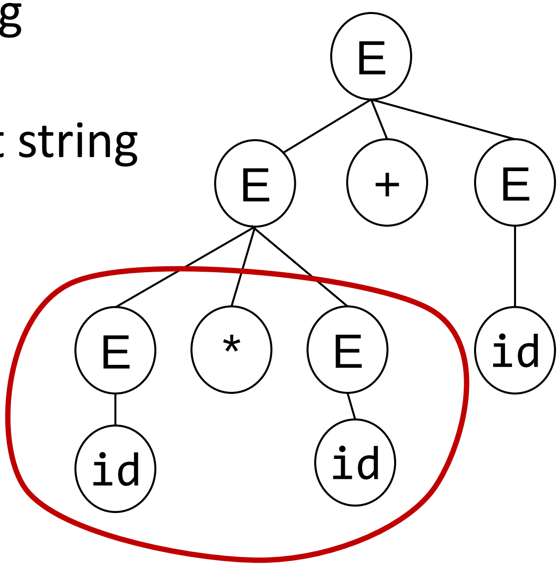
- Note in previous slides:
  - Replacement done on left-most non-terminal in the string - **called left-most derivation**
  - Terminals at leaves and non-terminal as interior nodes
  - `Inorder` traversal of leaves produces input string `id*id+id`



Derivations and Parse Trees

- Note in previous slides:
  - Replacement done on left-most non-terminal in the string - **called left-most derivation**
  - Terminals at leaves and non-terminal as interior nodes
  - `Inorder` traversal of leaves produces input string `id*id+id`
  - Parse tree shows association of operations. Input string doesn't
    - * associated with identifiers in the subtree

`(id * id)+id`



Derivations and Parse Trees

- Consider the same grammar (having the following rules):

```
1: E -> E + E
2:      | E * E
3:      | id
```

- Produce derivations for the string: **`id*id+id`**
  - Using **right-most derivations**

    i.e. replace the right-most non-terminal

Derivations and Parse Trees

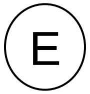
- Consider the grammar with the following rules:

```
1: E -> E + E
2:    | E * E
3:    | id
```

- Produce derivations for the string: **`id*id+id`**

Start with E, the start symbol

E



Parse Tree

Derivations and Parse Trees

- Consider the grammar with the following rules:

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1: E -> E + E
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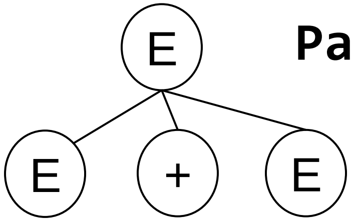
3:      | id
```

- Produce derivations for the string: **`id*id+id`**

`Apply 2:` **Replace E with E+E**

E

E+E



`Parse Tree`

Derivations and Parse Trees

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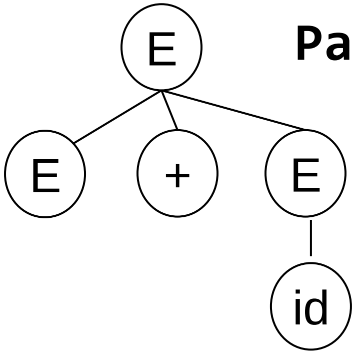
`Apply 1:` **Replace E with id**

```
E
E+E
E+id
```



Parse Tree

Derivations and Parse Trees

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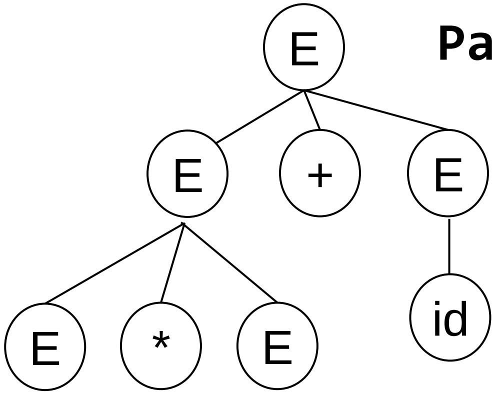
3:      | id
```

- Produce derivations for the string: **`id*id+id`**

`Apply 3:` **Replace E with E * E**

```
E
E+E
E+id
E*E+id
```



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Derivations and Parse Trees

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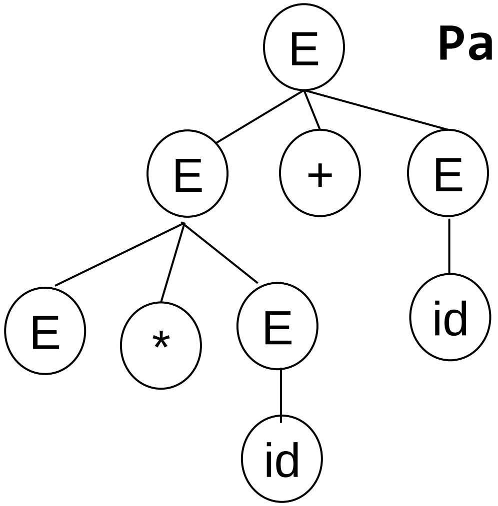
- Produce derivations for the string: **`id*id+id`**

`Apply 3:` **Replace E with id**

```
E
E+E
E+id
E*E+id
E*id+id
```



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Derivations and Parse Trees

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`Apply 3:` **Replace E with id**

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E+id
E*E+id
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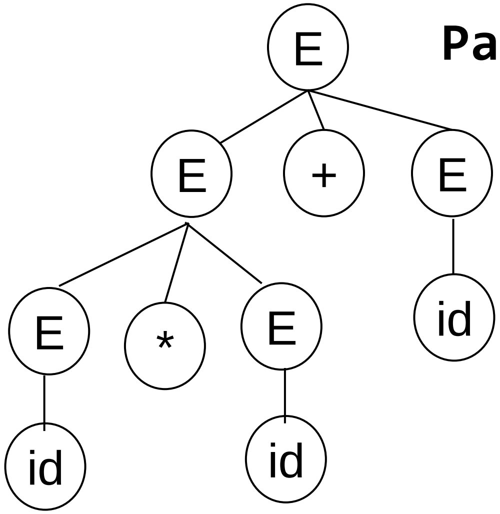
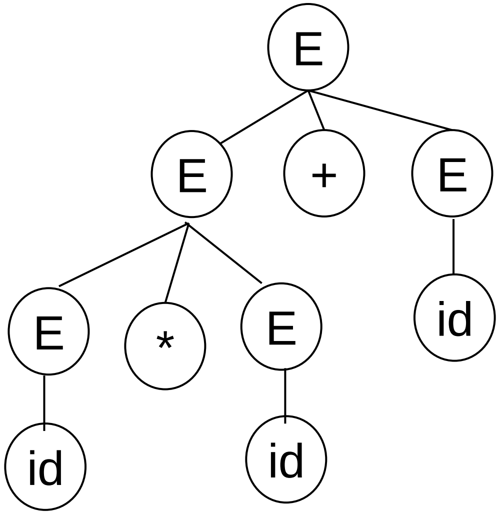
id*id+id
```



`Parse Tree`

Derivations and Parse Trees

- We get the same parse tree using left-most and right-most derivations.
  - Every parse tree has left-most and right-most (and any random order) derivations.



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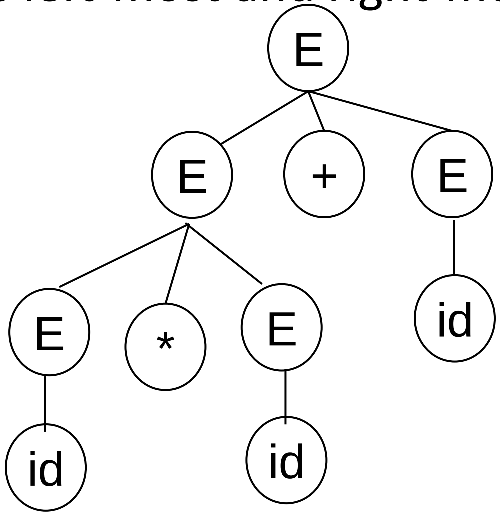

- But there could be a string (or more than one strings) for which there exists derivations that would get different parse trees

Derivations and Parse Trees

- Consider the grammar with the following rules:

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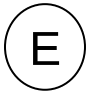
- Produce derivations for the string: **`id*id+id`**

Start with E, the start symbol

E



Parse Tree

Derivations and Parse Trees

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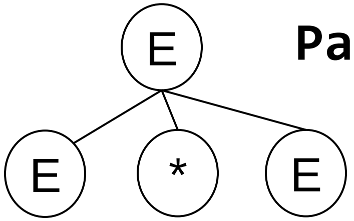
```

- Produce derivations for the string: **`id*id+id`**

Earlier it was replace E with E+E

`Apply 2:` **Replace E with E*E**

E
E*E



`Parse Tree`

Derivations and Parse Trees

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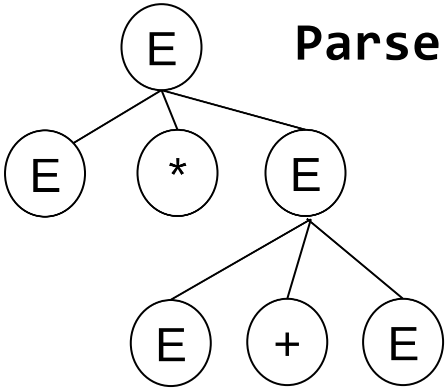
```

- Produce derivations for the string: **`id*id+id`**

`Apply 1:` **Replace E with E+E**

E
E*E
E*E+E



`Parse Tree`

Derivations and Parse Trees

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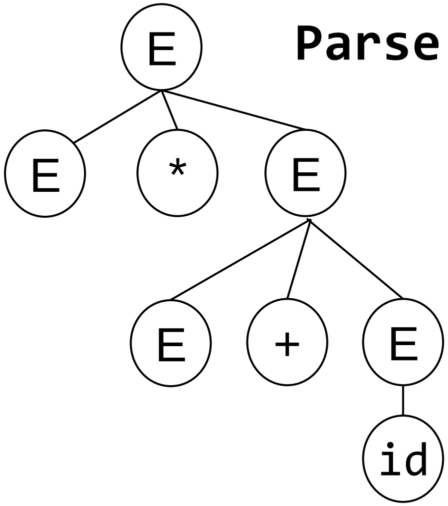
Apply 3: **Replace E with id**

E
E*E
E*E+E
E*E+id



Parse Tree

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Apply 3: Replace E with id

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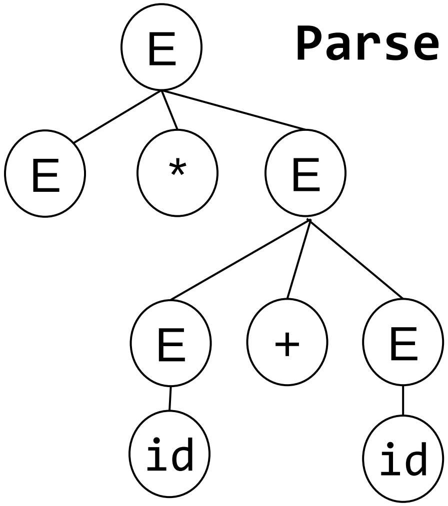

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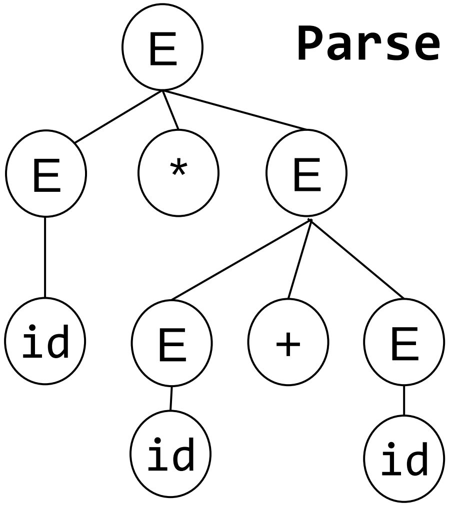
```

- Produce derivations for the string: **`id*id+id`**

`Apply 3:` **Replace E with id**

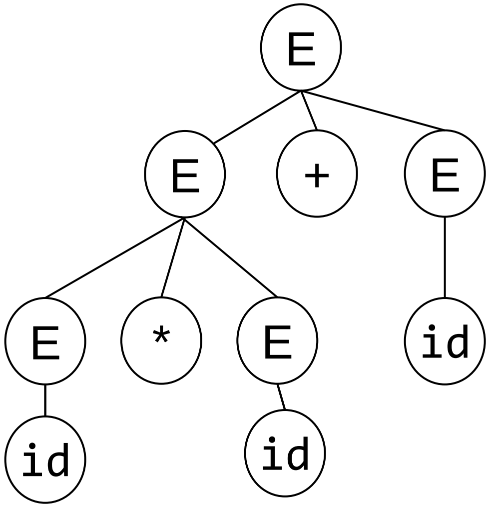
```
E
E*E
E*E+E
E*E+id
E*id+id
id*id+id
```



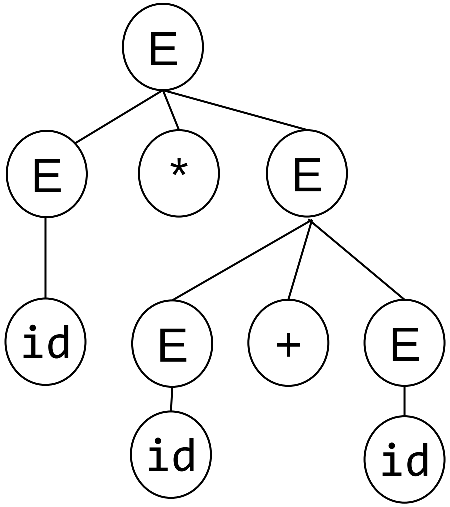
Parse Tree

Derivations and Parse Trees

- Input string: `id*id+id`



earlier



now

- `Inorder` traversal of leaves in both trees produces the same input string

Ambiguous Grammar

- Grammar that produces more than one parse tree for some string

```
1: E -> E + E
2:      | E * E
3:      | id
```

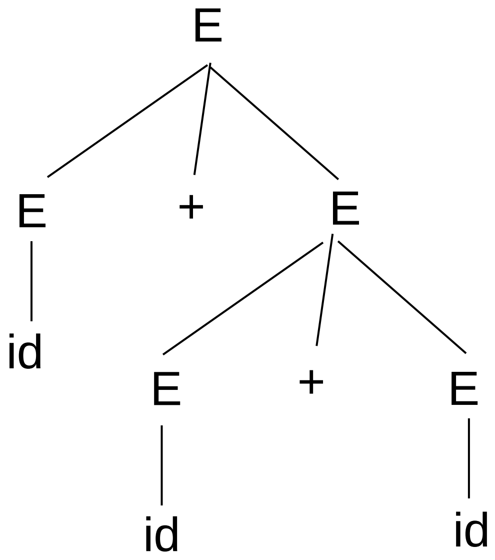
Ambiguity – what to do?

- Ignore it (let it be ambiguous)
  - Give hints to other components of the compiler on how to resolve it
- Fix it (Manually)
  - May make the grammar complicated and difficult to maintain

Ambiguity – ignore

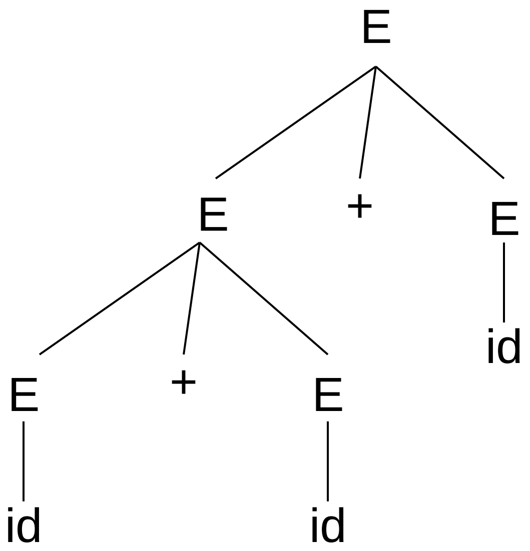
- Grammar: `E -> E + E | id` input:`id+id+id`

`E->E+E`
`E->id+E`
`E->id+E+E`
`E->id+id+E`
`E->id+id+id`



`Matches the input, which would be evaluated (later) as:` `id+(id+id)`

`E->E+E`
`E->E+E+E`
`E->id+E+E`
`E->id+id+E`
`E->id+id+id`



`(id+id)+id`

`%left +` **Provide hint (in Bison). Associativity declaration.**

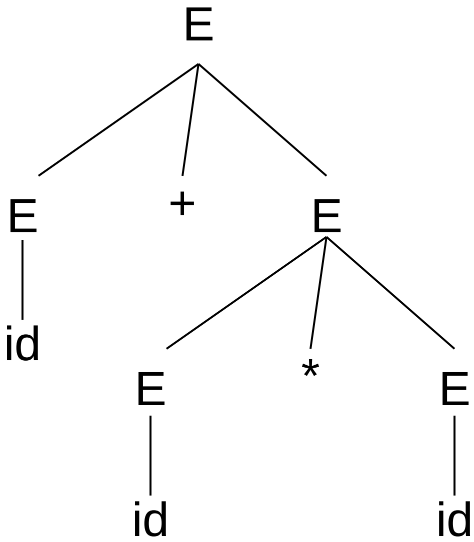
(left associative for +. So, produces the parse tree on the right)

Ambiguity - ignore

- `E -> E + E | E * E | id`

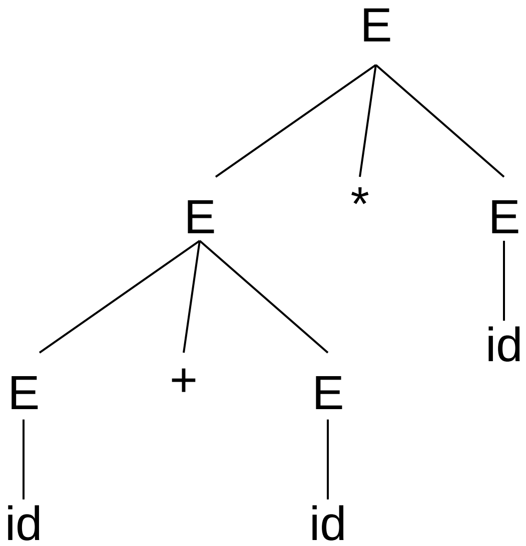
E->E+E
E->id+E
E->id+E*E
E->id+id*E
E->id+id*id

Produces
tree for:
id+(id*id)



E->E*E
E->E+E*E
E->id+E*E
E->id+id*E
E->id+id*id

Produces
tree for:
(id+id)*id



%left +
%left *

*Tells that * has higher precedence over + and both are left associative. So, we get the tree on left.*

Ambiguity – fixing

- Rewrite `E -> E + E | E * E | id` as:

```
E -> E + E
   | E * E
   | id
```

```
E -> E’ + E | E’
E’ -> id * E’ | id
    | (E) * E’ | (E)
```

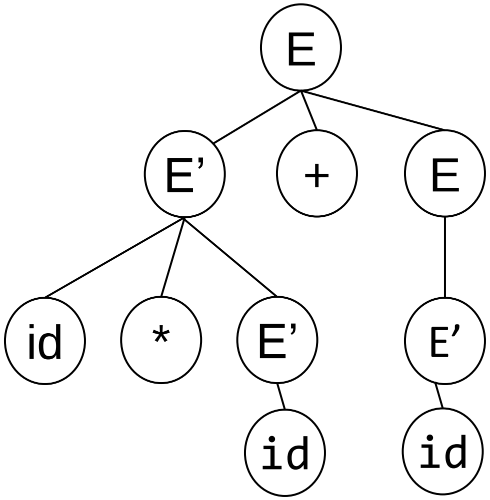
If you want to handle parenthesized expressions such as (id+id)*id

- E controls generation of +
- E’ controls generation of *

*’s are always nested deeper in the parse tree.

Parse tree for input id*id+id

```
E->E’+E
E’->id*E’
E’->id
E->E’
E’->id
```



is the above sequence left-most or right-most derivation?

Ambiguity Fixing - Exercise

Exercise: *Is this grammar ambiguous? Draw parse tree(s) for the following* **`input:`** **`if e1 then if e2 then s1 else s2`**

```
1: STMT -> if EXPR then STMT
2:       |  if EXPR then STMT else STMT
3:       |  s1
4:       |  s2
5: EXPR -> e1 | e2
```

Ambiguity Fixing - Exercise

Exercise: *Is this grammar ambiguous? Draw parse trees for the following* **input:** `if e1 then if e2 then s1 else s2`

```
1: STMT -> if EXPR then STMT
2:        |  if EXPR then STMT else STMT
3:        |  s1
4:        |  s2
5: EXPR -> e1 | e2
```

Ambiguity Fixing - Exercise

Exercise: *Is this grammar ambiguous? Draw parse trees for the following* **input:** `if e1 then if e2 then s1 else s2`

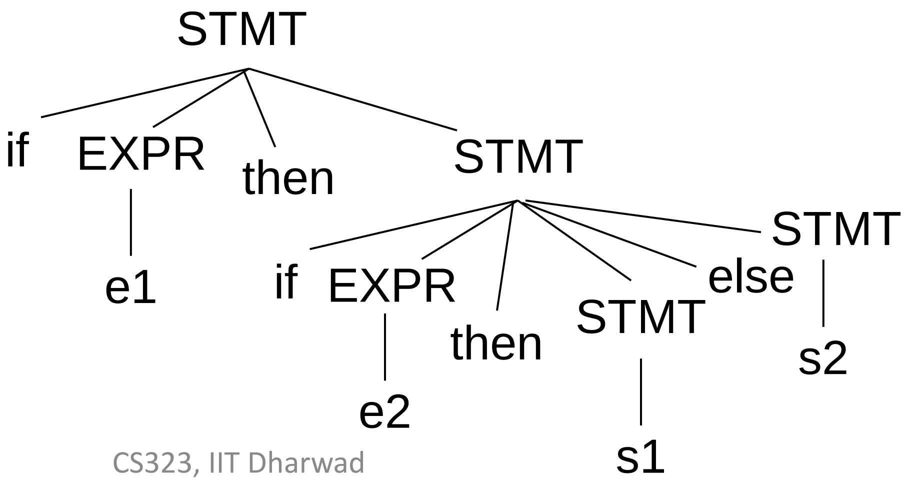
```
1: STMT -> if EXPR then STMT
2:        |  if EXPR then STMT else STMT
3:        |  s1
4:        |  s2
5: EXPR -> e1 | e2
```

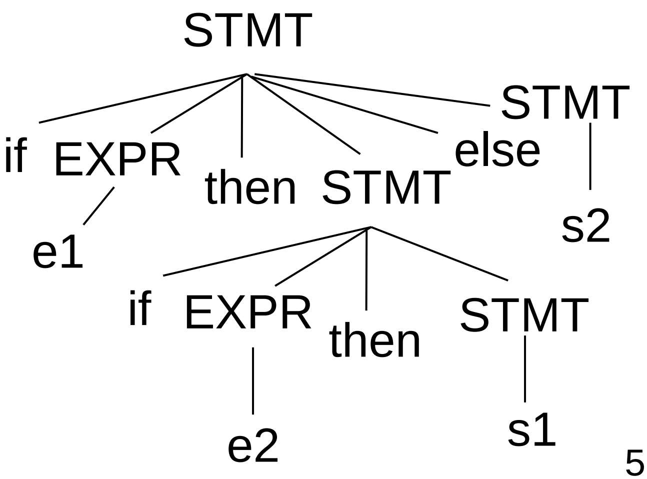
Ambiguity Fixing - Exercise

Exercise: *Is this grammar ambiguous? Draw parse trees for the following*

input: `if e1 then if e2 then s1 else s2`

```
1: STMT -> if EXPR then STMT
2:       |  if EXPR then STMT else STMT
3:       |  s1
4:       |  s2
5: EXPR -> e1 | e2
```

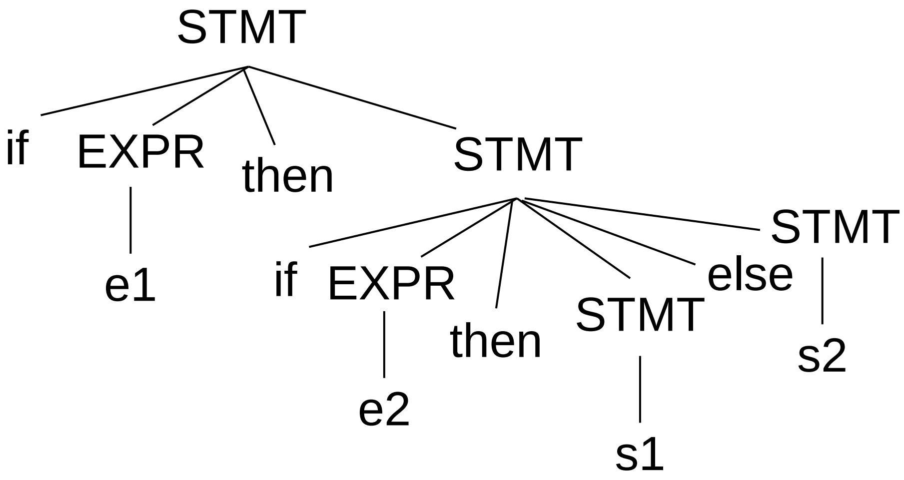




Ambiguity Fixing - Exercise

Exercise: Which `if` is the `else` associated with?

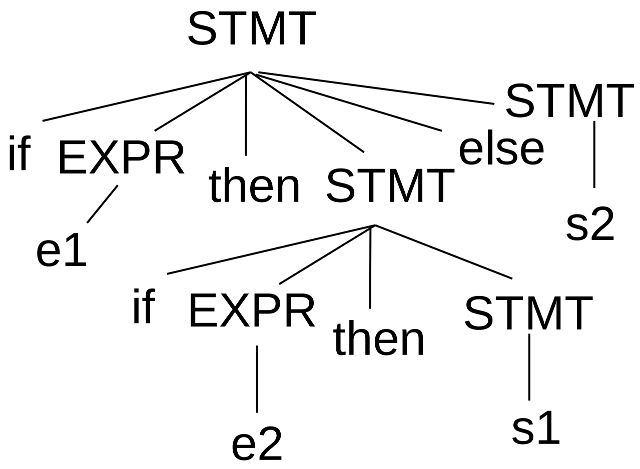
`String:` **`if e1 then if e2 then s1 else s2`**



Ambiguity Fixing - Exercise

Exercise: Which `if` is the `else` associated with?

`String:` **`if e1 then if e2 then s1 else s2`**



Ambiguity Fixing - Exercise

Exercise: Rewrite the grammar to make it unambiguous.

```
1: STMT -> if EXPR then STMT
2:       |  if EXPR then STMT else STMT
3:       |  s1
4:       |  s2
5: EXPR -> e1 | e2
```

CFG and Parsers

- Is it enough if parsers answer “yes” or “no” to check if a string belongs to context-free language?
  - Also need a parse tree
- What if the answer is a “no”?
  - Handle errors
- How do we implement CFGs?
  - E.g. Bison

Next

Error Handling

- Objective: detect invalid programs and provide meaningful feedback to programmer
    - Report errors accurately
    - Recover from errors quickly
    - Don’t slow down compilation

Error Types

- Many types of errors:
  - Lexical – `int 9abc;` //invalid identifier
  - Syntactic – extra brace inserted {
  - Semantic – `float sqr; sqr(2);`
    //use variable name with function call syntax
  - Logical – use = instead of ==

Error Handling - Types

1. Panic mode
2. Error production
3. Automatic local or global correction

Panic Mode Error Handling

- Simplest, most popular
- Discards tokens until one from a set of *synchronizing tokens* is found
- Synchronizing tokens have a clear role

  e.g. semicolons, braces
- E.g. i= i++j

  policy: while parsing an expression, discard all tokens until an identifier is found. *This policy skips the additional +*
- Specifying policy in bison: `error` keyword

  `E -> E + E | (E) | id | error id | error`

Error Productions

- Anticipate common errors
  - 2 x instead of 2 *
- Augment the grammar
  - E -> EE | …
- Disadvantages:
  - Complicates the grammar

Error Corrections

- Rewrite the program – find a “nearby” correct program
  - Local corrections – insert a semicolon, replace a comma with semicolon etc.
  - Global corrections – modify the parse tree with “edit distance” metric in mind
- Disadvantages?
  - Implementation difficulty
  - Slows down compilation
  - Not sure if “nearby” program is intended

Parsers – what do we need to know?

1. How do we define language constructs?
    - Context-free grammars
2. How do we determine: 1) valid strings in the language? 2) structure of program?
    - LL Parsers, LR Parsers

Next

3. How do we write Parsers?
    - E.g. use a parser generator tool such as Bison

Top-down Parsing

- Idea: we know sentence has to start with initial symbol
- Build up partial derivations by *predicting* what rules are used to expand non-terminals
  - Often called *predictive parsers*
- If partial derivation has terminal characters, *match* them from the input stream

Slide courtesy: Milind Kulkarni

Top-down Parsing

- Also called recursive-descent parsing
- Equivalent to finding the left-derivation for an input string
  - Recall: expand the leftmost non-terminal in a parse tree
  - Expand the parse tree in pre-order i.e., identify parent nodes before children

Top-down Parsing

↑: next symbol to
be read

```
1: S -> cAd
2: A -> ab
3:      | a
```

Step	Input string	Parse tree
1	cad ↑	S

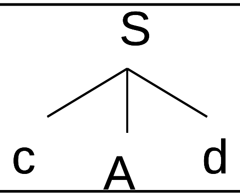
String: **cad**

Start with S

Top-down Parsing

↑: next symbol to be read

```
1: S -> cAd
2: A -> ab
3:      | a
```

Step	Input string	Parse tree
1	cad ↑	S
2	cad ↑	

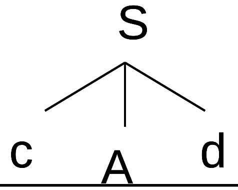
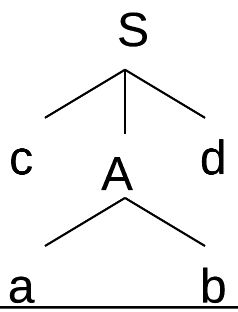
String: **cad**

Predict rule 1

Top-down Parsing

↑: next symbol to be read

1: S -> cAd
2: A -> ab
3: | a

Step	Input string	Parse tree
1	cad ↑	S
2	cad ↑	
3	cad ↑	

String: **cad**

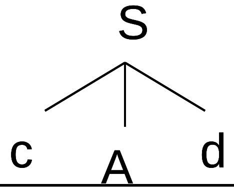
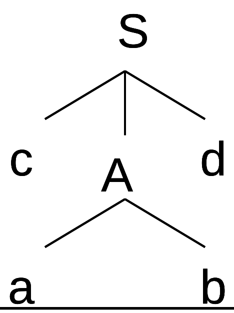
Predict rule 2

Top-down Parsing

↑: next symbol to be read

```
1: S -> cAd
2: A -> ab
3:      | a
```

String: **cad**

Step	Input string	Parse tree
1	cad ↑ (under c)	S
2	cad ↑ (under a)	
3	cad ↑ (under d)	

No more non terminals!

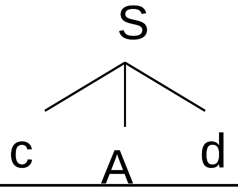
String doesn't match.

Backtrack.

Top-down Parsing

↑: next symbol to be read

```
1: S -> cAd
2: A -> ab
3:      | a
```

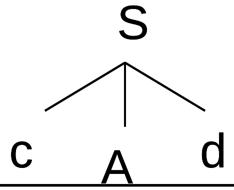
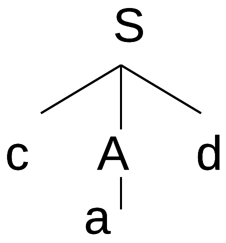
Step	Input string	Parse tree
1	cad ↑	S
2	cad ↑	

String: **cad**

Top-down Parsing

↑: next symbol to be read

```
1: S -> cAd
2: A -> ab
3:      | a
```

Step	Input string	Parse tree
1	cad ↑	S
2	cad ↑	
4	cad ↑	

String: **cad**

Predict rule 3

Top-down Parsing – Table-driven Approach

```
1: S -> F
2: S -> (S + F)
3: F -> a
```

string: `(a+a)`

string’: `(a+a)$`

	(	)	a	+	$
S	2	-	1	-	-
F	-	-	3	-	-

Assume that the table is given.

Top-down Parsing – Table-driven Approach

```
1: S -> F
2: S -> (S + F)
3: F -> a
```

string: `(a+a)`

string’: `(a+a)$`

	(	)	a	+	$
S	2	-	1	-	-
F	-	-	3	-	-

Assume that the table is given.

- Table-driven (Parse Table) approach doesn’t require backtracking

But how do we construct such a table?

Important Concepts: First Sets and Follow Sets

First and follow sets

- First(α): the set of terminals (and/or λ) that begin all strings that can be derived from α
  - First(A) = {x, y, λ}
  - First(xaA) = {x}
  - First (AB) = {x, y, b}
- Follow(A): the set of terminals (and/or $, but no λs) that can appear immediately after A in some partial derivation
  - Follow(A) = {b}

S → A B $

A → x a A

A → y a A

A → λ

B → b

Slide courtesy: Milind Kulkarni

First and follow sets

- $\text{First}(\alpha) = \{a \in V_t \mid \alpha \Rightarrow^* a\beta\} \cup \{\lambda \mid \text{if } \alpha \Rightarrow^* \lambda\}$
- $\text{Follow}(A) = \{a \in V_t \mid S \Rightarrow^+ \ldots Aa \ldots\} \cup \{\$ \mid \text{if } S \Rightarrow^+ \ldots A\ \$\}$

S: start symbol
a: a terminal symbol
A: a non-terminal symbol
α,β: a string composed of terminals and non-terminals (typically, α is the RHS of a production

$\Rightarrow$: derived in 1 step
$\Rightarrow^*$: derived in 0 or more steps
$\Rightarrow^+$: derived in 1 or more steps

Slide courtesy: Milind Kulkarni

Computing first sets

- Terminal: First(a) = {a}
- Non-terminal: First(A)
  - Look at all productions for A

    $A \rightarrow X_1 X_2 \dots X_k$
  - First(A) $\supseteq$ (First(X_1) - λ)
  - If $\lambda \in$ First(X_1), First(A) $\supseteq$ (First(X_2) - λ)
  - If λ is in First(X_i) for all i, then $\lambda \in$ First(A)
- Computing First(α): similar procedure to computing First(A)

Slide courtesy: Milind Kulkarni

Towards parser generators

- Key problem: as we read the source program, we need to decide what productions to use

- Step 1: find the tokens that can tell which production P (of the form A → $X_1X_2 \ldots X_m$) applies

$$\text{Predict}(P) =$$

$$\begin{cases} \text{First}(X_1 \ldots X_m) & \text{if } \lambda \not\in \text{First}(X_1 \ldots X_m) \\ (\text{First}(X_1 \ldots X_m) - \lambda) \cup \text{Follow}(A) & \text{otherwise} \end{cases}$$

- If next token is in Predict(P), then we should choose this production

First Set - Example

```
1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ
```

`first (S) = { ? }`

Think of all possible strings derivable from `S`.
Get the first terminal symbol in those strings
or λ if `S` derives λ

First Set - Example

```
1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ

first (S) = { x, y, c }
```

First Set - Example

```
1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ
```

```
first (S) = { x, y, c }
first (A) = {  ?  }
```

First Set - Example

1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ

first (S) = { x, y, c }
first (A) = { x, y, c }

First Set - Example

1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ

first (S) = { x, y, c }
first (A) = { x, y, c }
first (B) = { ? }

First Set - Example

1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ

first (S) = { x, y, c }
first (A) = { x, y, c }
first (B) = { b, λ }

Follow Set - Example

```
1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ
```

`follow (S) = { ? }`

Think of all strings **possible in the language** having the form `..Sa..` Get the following terminal symbol a after S in those strings or `$` if you get a string `..S$`

Follow Set - Example

```
1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ
```

```
follow (S) = {  }
```

Follow Set - Example

```
1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ

follow (S) = {   }
follow (A) = { ? }
```

Follow Set - Example

```
1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ
```

```
follow (S) = {   }
follow (A) = { b, c }        e.g. xaAbc$, xaAc$
```

Follow Set - Example

```
1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ
```

`follow (S) = {  }`

`follow (A) = { b, c }` e.g. `xaAbc$, xaAc$`

What happens when you consider: `A -> xaA or A -> yaA ?`

Follow Set - Example

```
1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ
```

```
follow (S) = {  }
follow (A) = { b, c }        e.g. xaAbc$, xaAc$
```

What happens when you consider: `A -> xaA or A -> yaA ?`

- You will get string of the form A=>$^{+}$ (xa)+A
- But we need strings of the form: `..Aa..` or `..Ab.` or `..Ac..`

Follow Set - Example

```
1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ
```

```
follow (S) = {   }
follow (A) = { b, c }
follow (B) = { ? }
```

Follow Set - Example

```
1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ
```

```
follow (S) = {   }
follow (A) = { b, c }
follow (B) = { c }
```

Predict Set - Example

1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ

$$\text{Predict}(P) =$$

$$\begin{cases} \text{First}(X_1 \ldots X_m) & \text{if } \lambda \notin \text{First}(X_1 \ldots X_m) \\ (\text{First}(X_1 \ldots X_m) - \lambda) \cup \text{Follow}(A) & \text{otherwise} \end{cases}$$

Predict (1) = { ? } = First(ABc$) if λ ∉ First(ABc$)

Predict Set - Example

1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ

	x	y	a	b	c	$
S	1	1			1	
A						
B						

Predict (1) = { x, y, c }

Predict Set - Example

1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ

	x	y	a	b	c	$
S	1	1			1	
A						
B						

Predict (1) = { x, y, c }
Predict (2) = { ? } = First(xaA) if λ ∉ First(xaA)

Predict Set - Example

1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ

	x	y	a	b	c	$
S	1	1			1	
A	2					
B						

Predict (1) = { x, y, c }
Predict (2) = { x }

Predict Set - Example

1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ

	x	y	a	b	c	$
S	1	1			1	
A	2					
B						

Predict (1) = { x, y, c }
Predict (2) = { x }
Predict (3) = { ? } = First(yaA) if λ ∉ First(yaA)

Predict Set - Example

1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ

	x	y	a	b	c	$
S	1	1			1	
A	2	3				
B						

Predict (1) = { x, y, c }
Predict (2) = { x }
Predict (3) = { y }

Predict Set - Example

```
1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ
```

	x	y	a	b	c	$
S	1	1			1	
A	2	3				
B						

```
Predict (1) = { x, y, c }
Predict (2) = { x }
Predict (3) = { y }
Predict (4) = { ? }
```

= First(c) if λ ∉ First(c)

Predict Set - Example

1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ

	x	y	a	b	c	$
S	1	1			1	
A	2	3			4	
B						

Predict (1) = { x, y, c }
Predict (2) = { x }
Predict (3) = { y }
Predict (4) = { c }

Predict Set - Example

1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ

	x	y	a	b	c	$
S	1	1			1	
A	2	3			4	
B						

Predict (1) = { x, y, c }
Predict (2) = { x }
Predict (3) = { y }
Predict (4) = { c }
Predict (5) = { ? }

= First(b) if λ ∉ First(b)

Predict Set - Example

1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ

	x	y	a	b	c	$
S	1	1			1	
A	2	3			4	
B				5		

Predict (1) = { x, y, c }
Predict (2) = { x }
Predict (3) = { y }
Predict (4) = { c }
Predict (5) = { b }

Predict Set - Example

1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ

	x	y	a	b	c	$
S	1	1			1	
A	2	3			4	
B				5		

Predict (1) = { x, y, c }
Predict (2) = { x }
Predict (3) = { y }
Predict (4) = { c }
Predict (5) = { b }
Predict (6) = { ? }

$$\text{Predict}(P) = \begin{cases} \text{First}(X_1 \ldots X_m) & \text{if } \lambda \notin \text{First}(X_1 \ldots X_m) \\ (\text{First}(X_1 \ldots X_m) - \lambda) \cup \text{Follow}(A) & \text{otherwise} \end{cases}$$

= First(λ) ?

Predict Set - Example

1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ

	x	y	a	b	c	$
S	1	1			1	
A	2	3			4	
B				5		

Predict (1) = { x, y, c }
Predict (2) = { x }
Predict (3) = { y }
Predict (4) = { c }
Predict (5) = { b }
Predict (6) = { ? }

$$\text{Predict}(P) = \begin{cases} \text{First}(X_1 \ldots X_m) & \text{if } \lambda \notin \text{First}(X_1 \ldots X_m) \\ (\text{First}(X_1 \ldots X_m) - \lambda) \cup \text{Follow}(A) & \text{otherwise} \end{cases}$$

= $\cancel{\text{First}(\lambda)}$? Follow(B)

Predict Set - Example

1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ

	x	y	a	b	c	$
S	1	1			1	
A	2	3			4	
B				5	6	

Predict (1) = { x, y, c }
Predict (2) = { x }
Predict (3) = { y }
Predict (4) = { c }
Predict (5) = { b }
Predict (6) = { c }

Computing Parse-Table

1) S -> ABc$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ

	x	y	a	b	c	$
S	1	1			1	
A	2	3			4	
B				5	6	

first (S) = {x, y, c}
first (A) = {x, y, c}
first(B) = {b, λ}

follow (S) = {}
follow (A) = {b, c}
follow(B) = {c}

P(1) = {x,y,c}
P(2) = {x}
P(3) = {y}
P(4) = {c}
P(5) = {b}
P(6) = {c}